

INTEREST RATES AND  
THEIR FORECASTING

An Honors Thesis (ID 499)

by

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## INTRODUCTION AND ECONOMIC BACKGROUND

The need for interest rate forecasts is more prevalent now than ever before in the insurance industry. In the past twenty years, interest rates have become much more volatile, which increases the risk that insurance companies are exposed to. Insurance companies have also introduced products which are more interest sensitive and are competing more closely with traditional financial securities. This forces insurance companies to price their products more competitively and with less "spread" so that the room for error is greatly diminished.

There are a number of ways to generate interest rate forecasts. Interest rates are affected by many influences, but probably most importantly by the Federal Reserve (Fed) policy. When Congress, or the President feels that interest rates should be lower (they never seem to want them higher!) they always put pressure on the Fed. However, the Fed does not set interest rates; interest rates are determined by supply and demand in the credit markets. The Fed is limited to three policy tools. First, it can change the discount rate, or the rate at which commercial banks can borrow from the Fed's "discount window." If they increase the discount rate, this will lower the money supply, and if they decrease the discount rate, it will increase the money supply. Secondly, they can change the reserve ratio, that is, the

percentage of deposits which banks must keep on reserve with the Fed. This tool works in the same manner as the discount rate in that it varies inversely with the money supply so that when they increase the reserve ratio, the money supply falls, and vice-versa. Finally, they can go out into the open market and buy or sell government securities. When the Fed buys securities, they pay for them using Federal Reserve notes, thus increasing the money supply. When they sell securities they are paid with their own notes, thus decreasing the money supply. All these policy tools affect the money supply which presumably affect interest rates. In what way they affect interest rates depends on what economic theory you choose to follow.

Economists also use leading economic indicators to try to predict how the economy will behave in the future. However, economists' performance has been far less than acceptable. The Blue Chip Economic Indicators are a poll of predictions from a large number of economists. In 8 of the past 11 years, their predictions have been wrong. Another study, "showed the top 4 prognosticators to be wrong about whether rates were going up or down -- never mind by how much -- a third of the time." (Nasar, 1990) The Wall Street Journal, also publishes a survey of interest rate forecasts made by economists from across the nation twice a year. The variance in the forecasts is striking, reflecting all the

different assumptions and theories that each economist ascribes to.

There are many explanations for this failure of economists. Interest rates depend on accurate forecasts of a myriad of factors including such diverse information as the inflation rate in the U.S.; foreign countries' stock markets; trade negotiations; the price of the dollar; investor sentiment, not just in America, but around the globe. The list could go on ad infinitum if one were to exhaust all the possible influences that affect the credit markets. The U.S. depends heavily on foreign capital to finance our huge deficit. If the Japanese were to decide to invest elsewhere, possibly because another country was offering higher interest rates, we would most likely have to raise ours in order to attract investors back to the U.S. For all these reasons, accurately forecasting interest rates using only economic theory can quickly become impractical.

Why use a random walk to generate future interest rates? While it at first appears that randomly generated interest rates would not be an appropriate method for attempting to deal with the future, there is actually a great deal of theoretical support for the idea.

#### RANDOM INTEREST RATES

Interest rates are determined in the credit markets where they are constantly fluctuating as new information

becomes available. Take for example U.S. Treasury Bills. We can assume that the only way their price (or yield) can change is by a change in the basic underlying factors which determine price such as risk, expected future interest rates, the prices of other securities, etc. We can also assume that all publicly available information concerning these basic factors is currently reflected in the price of the security. This describes what economists call an "efficient market," one in which all current information is almost immediately reflected in the current price. The only way these factors can change is by new information becoming available, which by definition cannot be forecast. Because the price can only change as a result of a "random" event of new information becoming available, a random walk becomes the only way to effectively forecast the price of securities.

The paper Interest Rate Scenarios by Merlin F. Jetton describes a number of different methods for creating interest rate scenarios which can be used to analyze an insurance company's portfolio.

I took these mathematical models and wrote programs, using the Pascal programming language, which would generate scenarios based upon the models. In comparing these models a number of measurements were made. First, one can simply look at the frequency of certain interest rates: that is, the number of times that interest rate occurred throughout

the different scenarios. These graphs appear in the Appendices 2.1 - 2.4.

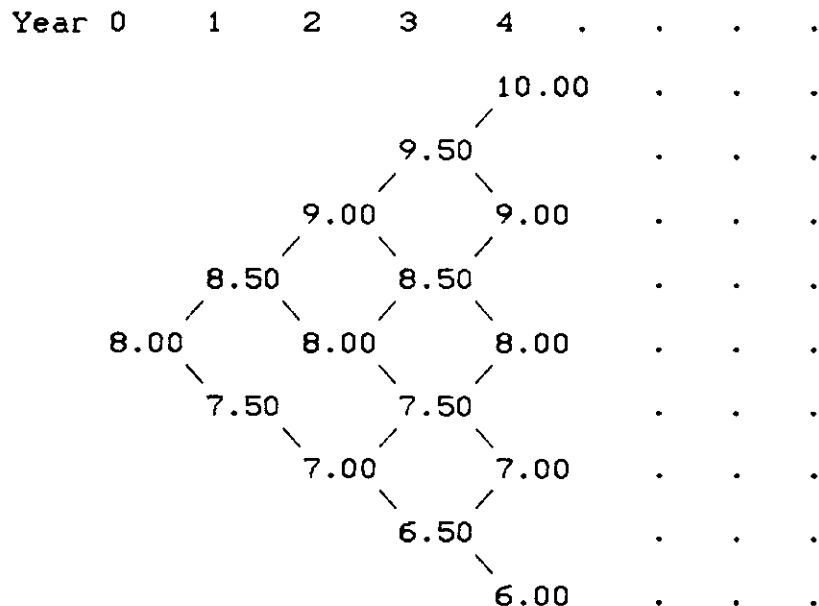
Secondly, an interesting measure of the model is to assume that one dollar was invested in the first year, and then accumulated at the randomly generated interest rates over the next years. One problem with the indicator is that regardless of what order the interest rates occur in, the result will be the same. For example, if over a three year period the random rates were determined to be 8.00%, 8.50% and 9.00%, the resulting accumulation would be equal even if the rates occurred in the order 8.50%, 9.00% and then 8.00%. This is fine for cases where you will buy and hold over the period of the scenario, but insurance companies would have a constant inflow and outflow of money, which would be sensitive to the timing of the interest rates.

To answer the above problem, the value of an annuity in which one dollar is invested at the beginning of each year of the scenario is considered. Thus, higher rates in the later years result in a larger accumulation than if the rates are lower in the later years. Once this accumulation is calculated, the constant rate that would have resulted in the same accumulation is calculated using a bisection method. All these measures are given in the appendices for each of the models used.



# BINOMIAL LATTICE MODEL

The first of these models is a simple binomial lattice model. With this model we start with a given interest rate, presumably the current rate, and then have a 1/2 probability of the interest rate increasing by a set amount, and a 1/2 probability of the rate falling by the same set amount. In the programmed model, I used an initial rate of 8.00% and a step of 0.50%. That is starting at 8.00%, one year later, the interest rate will be either 8.50% or 7.50%. The following year the possibilities expand to 9.00%, 8.00%, and 7.00%, obviously limited by the realized rate of the previous year.



The results of running this model, compared with the basic third stochastic model, are given in Appendix 2.4. As one immediately can see this model has a number of

limitations. First of all the change from year to year is limited to the step size which was chosen by the modeler. Secondly, it is possible for interest rates to get far away from historical norms. For instance, with this model the interest rates could actually fall below 1.00% or even below 0, as well as climb to rates higher than what has ever been experienced in the past.

This is by far the easiest model to understand. The modeler simply starts at the present interest rate, then flips a coin. If it's heads, we assume interest rates rise by 1/2%, tails, they fall by 1/2% in the next year. He then starts at the newly generated rate and again flips the coin, using the same rule as in the previous year to determine whether rates rise or fall. He continues this process until he has generated the selected number of future interest rates.

## SECOND STOCHASTIC MODEL

The second model makes a number of improvements over the binomial lattice. First, it allows the interest rate to change by different amounts each year. This model can be summarized by the following formula:

$$I_{t+1} = I_t e^{(Z \times VF)}$$

Where

$I_t$  = one-year rate at time  $t$

$Z$  = random number from normal distribution

VF = volatility factor of one-year interest rate changes

To get next years rate ( $I_{t+1}$ ) you take this years rate ( $I_t$ ) and multiply by  $e$  raised to the power of  $Z \times VF$ . Because  $Z$  is randomly chosen from the normal distribution, the interest rate will change by  $\pm 1$  standard deviation (volatility factor) in 96% of the cases, while it still has the ability to change by more than that in extreme cases. For example, say  $Z=0$ , then the interest rate would not change from year  $t$  to year  $t+1$ . If  $Z=2$ , and the volatility factor was 0.23, then next years interest rate would be 1.584 times last years rate.

The assumption with this model is that  $\ln[(I + dI)/I]$  is normally distributed, where  $dI$  is equal to the change in  $I$ , or  $(I_{t+1} - I_t)$ . Using historical one-year Treasury interest rates Mr. Jetton calculated the standard deviation of this measure to be 0.23. I used the one-year Treasury rates as published by the Federal Reserve Bulletin over the year 1960 to 1989 and calculated the standard deviation of the year-to-year volatility to be 0.2022. I also calculated the standard deviation of this measure using Yields on Government Securities compiled by Salomon Brothers. (Strommen, 1989) This standard deviation was 0.2271. The raw data from both these sources may be found in Appendix 1.1 and 1.2.

The second stochastic model still contains some problems. While the volatility over one-year periods is accurate, the randomly generated interest rates are much more volatile than the historical rates over long periods of time. As with the binomial lattice model, interest rates can rise far above historical rates, or fall far too low. Mr. Jetton suggest two possible changes to the model in order to avoid these problems. The first is to lower the volatility factor. This would solve much of the problem. However, if you are interested in fluctuations over one-year periods, as most businesses are, the volatility would be too low. The second solution is to put lower and upper bounds on the interest rates. This would also solve the first problem, but would cause another problem in that the rates can get stuck at the lower or upper bound when they can only move in one direction, or stay the same. Finally, one could use both of the above solutions, but this would still cause the same problems that using them individually caused. These two modifications were made and the results of the random rates are summarized in Appendix 2.3 and 5.1.

### THIRD STOCHASTIC MODEL

The third model that Mr. Jetton discusses seeks to remove the problems that arose in using the second model. It also is extended to generate a wide range of interest rates rather than just one rate. First a one-year rate is

generated, followed by the twenty-year rate. Finally the intermediate rates are found by interpolation between the one and twenty year rates to generate a complete yield curve.

In generating the one-year Treasury rate, recall that we wanted to retain the volatility in short-run changes in the rate, yet in the long-run we did not want to allow the volatility to exceed historical standards. Mr. Jetton's proposal for alleviating this problem is to use a mean-reverting formula. A mean-reverting formula attempts to pull the interest rate back down to the mean, or expected interest rate, when it gets too far out of line. The calculation for determining the one-year rate is as follows:

$$T_1(t+1) = [T_1(t) + f(t)]e^{(2 \times VF)}$$

where

$f(t)$  = minimum of

$$[ 0.015(d^3) , 0.5(d) ]$$

for  $T_1(t) < T_1(\infty)$

= maximum of

$$[ 0.015(d^3) , 0.5(d) ]$$

for  $T_1(t) \geq T_1(\infty)$ ;

where

$$d = T_1(\infty) - T_1(t)$$

$T_1(t)$  = one-year rate in year  $t$

$T_1(\infty)$  = the expected long-run one-year rate

VF = volatility factor as used in the last model

$Z$  = random number from normal distribution

The function  $f(t)$  has a number of characteristics worth discussing. First of all it has more effect on the interest rates the further they move away from the long-run rate. This reflects the feeling that if the market feels that the current rates are higher or lower than what are perceived to be "normal," then the market forces will work to bring them back into line. One should note that the interest rates used in this calculation are assumed to be in percentage form rather than decimal form. For example, it would use 8.00 rather than 0.08. At first, it may appear that this formula will always select the cubed part of the formula rather than the linear part. "That is largely but not entirely correct. 'Always' should be 'generally.' ... Let  $d = |T_1(\infty) - T_1(t)|$ . Then the cubic part will hold if  $0.015d^3 < 0.5d$ , i.e. if  $d < 5.77$ . Therefore, the linear part of the formula will come into play only when  $T_1(t)$  is very high or very low." (Jetton, 1991) One problem with

this function is in the setting of  $T_1(\infty)$ . The mean-reverting function will force the anticipated future rate to move toward this rate, yet no one can know for certain what future interest rates will trend toward. Possible solutions to this problem are discussed later in this paper.

The benefits of using this mean-reverting formula over the use of upper and lower bounds as in the previous model are quite obvious. In the case of the bounds, the interest rates remained unaffected until they hit the upper or lower bound. That is, they do not pull the interest rate toward the normal rate, but rather simply stop the rate from increasing or decreasing any further once a bound has been hit. With the above mean-reverting function, rates are constantly being pulled back to their long-term expectation, and the intensity with which they are pulled increases proportionally with the distance that they are removed from their normal rate.

#### MODIFICATIONS TO THIRD STOCHASTIC MODEL

In the discussions which follow Mr. Jetton's paper, a number of possible modifications and improvements were given. There were also a number of questions raised by the discussants. It is interesting to note that the number of discussants to the paper was a record of nine, rather than the usual two or three. This shows, as Mr. Jetton thought,

that there are a great number of actuaries interested in interest rate scenarios, many more than what would be indicated by the literature available on it.

Mr. Buff and Mr. Lassow ask a number of questions in their discussion of the paper. I will attempt to answer many of them. Their first question is "How different are the interest rate scenarios based on the third stochastic model compared to those based on the second stochastic model? More importantly, how different is the range of output financial results based on the two models?" The graph in Appendix 2.2 shows the frequency of different interest rates between the two models. As one can see, the third stochastic model is much more clustered around the initial rate than is the second. The standard deviation of the second model's average rate is over four times as large as the third's. The mean rate of the second model is 12.05%, while it is very close to  $T_1(\infty)$  for the third model (8.17%).

As the above question stated, even more important than the interest rates generated, is the financial results which are derived from assuming these rates occurred. In this case the difference is very striking. The second stochastic model is very far from being anywhere near what one would expect. An average accumulated value of \$1 reaching \$104 in twenty years is obviously outrageous. Just as Mr. Jetton



explains, the volatility factor can cause larger problems the longer the scenario lasts. For example:

$$I_{t+n} = I_t e^{VF(Z_1 + Z_2 + Z_3 + \dots + Z_n)}$$

For example, if  $I_t = 8$  percent and  $\sum Z_k = 5$ , then  $I_{t+n} = 25.26$  percent! The probability that this occurs over a twenty year scenario is 0.1335. ( $\sum Z_k$  is distributed normally with mean 0 and variance 20). The examining reader will note that in Appendix 2.2 and Appendix 5.1 two different runs are made using the second stochastic model with the exact same assumptions. However, the results are markedly different. This can be attributed only to the randomness involved in the model, and should make the user even more aware of the fact that he must be very careful in using the results from such a volatile model.

One should also note that the standard deviations are much greater than the means from the second model. This results from the fact that over the 100 scenarios generated, there were a great many outlying observations. That is, a number of times the model generated interest rates which would allow accumulations to be completely out of line with historical or even common sense standards.

#### ALTERNATE MEAN REVERTING FORMULAS

On page 439, Mr. Strommen offers a possible modification to the mean-reverting formula used in the third

model. He identifies the main problem with the original formula is in the choosing of  $T_1(\infty)$ . When interest rates are higher than  $T_1(\infty)$ , they are pulled down, and when they are lower than  $T_1(\infty)$ , they are pushed up. If one chooses a value for  $T_1(\infty)$  which is different from the current interest rate, this will introduce an upward or downward bias into every scenario generated. While this may be desirable for someone with prior expectations as to the direction in which future interest rates will move, as was discussed at the beginning of this paper, it is very hard to support any prediction, and even the finest economists are regularly unable to choose correctly.

Mr. Strommen's alternative approach is to assume that there is a range of normal values for  $T_1(\infty)$  rather than a single rate. Mr. Jetton's basic formula is still used, however, it is set equal to zero whenever  $T_1(t)$  is within the assumed normal values. For example, if we choose our normal range of value to be between 4 and 10 percent, then the new mean-reverting formula can be described by the following:

$$f(t) =$$

minimum of

$$[ 0.015(d^3) , 0.5d ]$$

$$\text{If } T_1(t) < T_1(\infty) \text{ AND } T_1(t) < 4.00$$

maximum of

$$[ 0.015(d^3) , 0.5d ]$$

$$(.015[T_1(\infty) - T_1(t)]^3; .5[T_1(\infty) - T_1(t)])$$

$$\text{if } T_1(t) \Rightarrow T_1(\infty) \text{ AND } T_1(t) > 10.00$$

Otherwise  $f(t)=0$

Where  $d = T_1(\infty) - T_1(t)$ .

This formula no longer requires the modeler to choose a single normal value for the interest rates, and allows him to choose a range of normal values. This approach has a number of advantages. First of all, because  $T_1(\infty)$  need not be set to the current interest rate, the model will not need constant modification as the current interest rate changes. Secondly, it will be easier to support your decision for a range of values, and will be accepted by more people.

The results of running this model against the model with the original mean-reverting formula are given in Appendix 4.1.

In his discussion, Mr. Gurski also offers his own slight modification to the mean-reverting formula. It can be summarized by the formula below:

$$T_{1adj}(t+1) =$$

$$T_1(t+1) + C * [T_1(\infty) - T_1(t+1)]$$

where  $C$  = a "central tendency" factor

Mr. Gurski suggests a value of 0.01 for  $C$  when projecting rates month to month. "This mean reverting formula has the desirable property of creating a cyclical pattern of interest rates. For example, as rates get high relative to their historical means, there is a fairly strong influence bringing the rates down (because the adjustment is larger as the rates get further from their mean.)" (Gurski, 1987) These results are also summarized in Appendix 4.2. For further discussion of the central tendency factor, see the section below.

In a personal letter, Mr. Jetton himself offered a further modification to the formula.

In the discussion of my paper on page 458, Graham Lord made about the same observation you did. My response to him at the time (page 476) was that I thought it would be difficult to base the formula on historical interest rates, for it would be difficult to distinguish between interest rate movements that are random and those that are due to mean reversion.

I have since concluded that it is not so difficult and believe that a more appropriate formula would be:

$$0.1d + 0.008d^3 \text{ but not } > 0.4d$$

This formula is more powerful than the one on page 431 for  $d < 3.78$ . It was based on average historical rates one year later, given the interest rate was at a prescribed level at the start of the one-year period...In my opinion that

[0.5d] would be far too powerful for small 'd'. (Jetton, 1991)

In his formula,  $d = |T_1(\infty) - T_1(t)|$ . The results of this modification are summarized in Appendix 4.3. The reason this formula is more powerful, is that it will bring interest rates back into line with the historical mean ( $T_1(\infty)$ ) faster than would the other formula.

#### THE CENTRAL TENDENCY MEASURE AND MONTHLY INTEREST RATES

Mr. Jeffrey Gurski gave a slightly different model to be used than did Mr. Jetton. He used a "central tendency" factor "C" which I questioned him about:

In regard to the mean reverting process I describe in my TSA XL discussion:

The methodology was chosen so as to simultaneously keep the rates reasonable in relation to historical precedents, and create a cyclical pattern justifiable by the notion of a business or economic cycle.

The actual value that I found for C was found using trial and error, and judgment. In my judgment, rates much above 20% are "unreasonable" for the U.S. One could argue for other such limits, I suppose. However, based on my judgment, I would try different values of C and project the rates month by month over a 5 year horizon. Perhaps 50 or 100 paths of yield curves would be run and inspected. If only a few rose above 20%, and only for a short period, I knew I had a good value for C.

If you are projected [sic] rates year to year, you may want to use a value of C

on the order of .12, but you should vary this depending on the strength of the mean reverting process you desire, the purpose the rates will be used for, and samples of the resulting rates.

Note that I used a similar methodology to keep the slope of the yield curve reasonable in relation to historical standards. (Gurski, 1991)

As Mr. Gurski suggests determining a value for C requires personal judgment and trial and error. You must decide what you feel are reasonable interest rates, and adjust the central tendency factor until you are comfortable with the interest being generated. Mr. Gurski also sent this author a very complete list of interest rates which could be used for further research. This list give monthly interest rates for 3 month, 6 month, 1 year, 2 year, 3 year, 5 year, 7 year, 10 year, 20 year, and 30 year Treasuries for the year 1954 to 1990. These are included in original form at the end of this paper.

#### SENSITIVITY TESTING

The first sensitivity test I carried out was on the volatility factor used in the Second Stochastic Model. As expected, Appendix 5.1 shows that the standard deviations have dropped considerably in all of the financial measures.

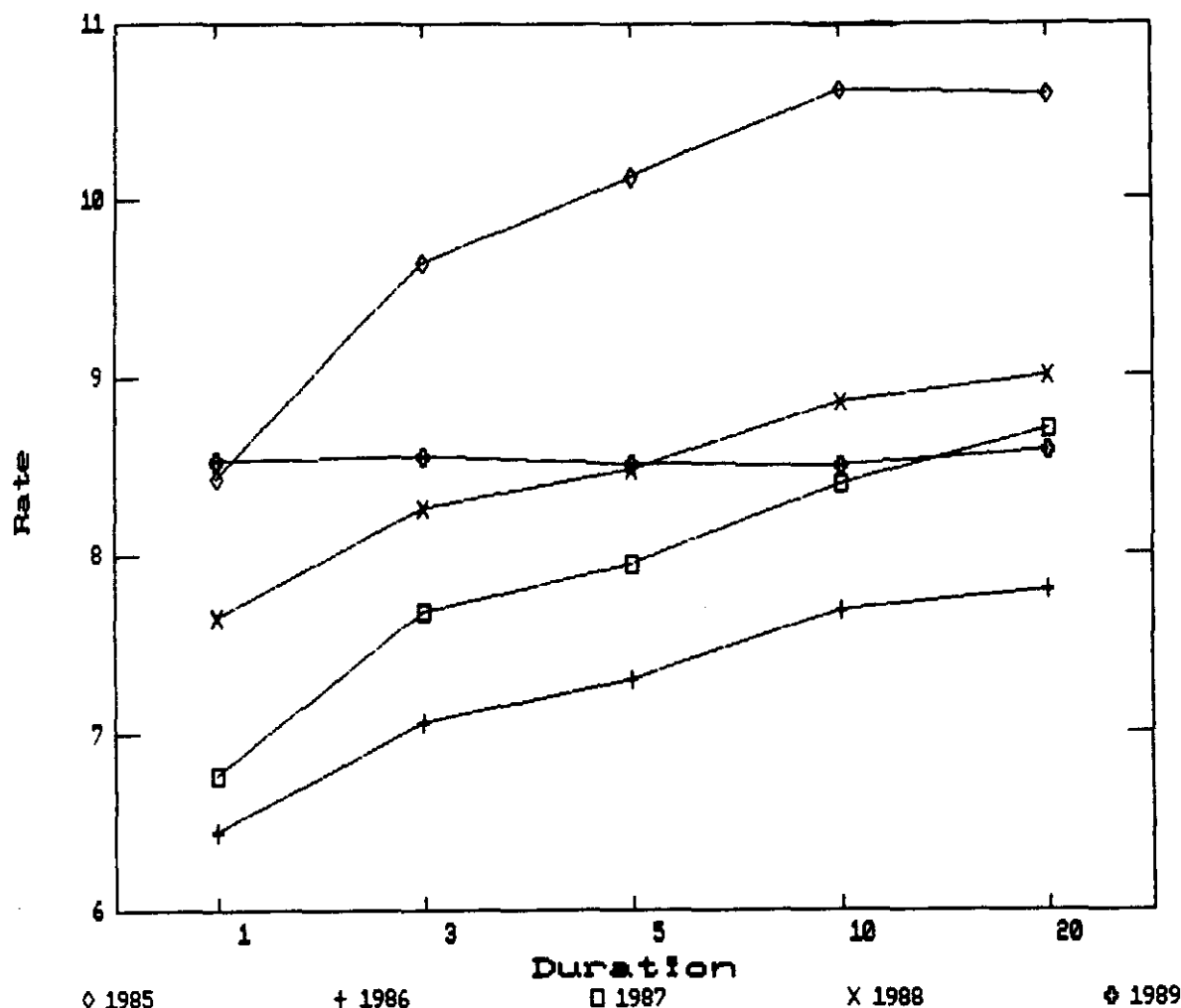
The second change I made was in the Third Stochastic Model, measuring the effect of changing  $T_1(\infty)$  by plus or minus 1.00%. Appendix 5.2 shows the graphs of each of these

changes. Quite simply, increasing  $T_1(\infty)$  shifts the entire graph to the right 1.00%, and to the left 1.00% when  $T_1(\infty)$  decreases.

#### HISTORICAL INTEREST RATES AND THE YIELD CURVE

Using historical interest rates from the twenty year period 1970-1989, the same measures used with the models were calculated. (see Appendix 3.3) One dollar invested in 1970, would have grown to \$4.89 by 1989. One dollar invested each year since 1970, would have accumulated to \$53.95 by 1989. The same would have resulted from a constant rate of 8.73%. The distribution of the rates over the past twenty years looks fairly close to that generated by the Third Stochastic Model. Our choice of 8.00% for  $T_1(\infty)$  is further justified by the fact that the average rate over the twenty years was 8.73%.

A yield curve shows the relationships between the different yields available for the various durations. The graph below shows examples from five different years. A "normal" yield curve slopes upward, that is, the yields for longer durations are higher than the yields for shorter durations. If the yield curve slopes downward it is called an "inverted" yield curve. The yield curve for 1989 is almost an inverted yield curve, but more accurately would be called a flat yield curve.



In his article, Mr. Strommen calculated a formula for predicting the slope of the yield curve for the current year given the slope from the previous year. The formula is given below:

$$Y = c + aX_1 + bX_2$$

where  $Y$  = slope of the yield curve

$$= S(t+1) = [T_{20}(t+1) - T_1(t+1)]/T_1(t+1)$$

$$X_1 = S(t)$$

$$X_2 = [T_1(t+1) - T_1(t)]/T_1(t)$$



He used data current up to 1984. When I contacted him he was kind enough to update this information to 1989. Using the additional data, I recalculated the regression. There was not a great change given the additional data, but the  $R^2$  did change significantly. The constant "c" went from 0.064 to 0.0669. The a parameter fell from 0.718 to 0.627. The "b" coefficient in the regression function changed from -0.587 to -0.569.  $R^2$  fell from 0.934 to 0.846. This is disappointing because the fit of a model is said to be "better" the closer  $R^2$  is to 1. If it is 1, it means that the fitted regression line explains 100 percent of the variation in Y. More detailed statistics from this regression are given in Appendix 3.1b, while the raw data is illustrated in Appendix 3.1a. A more detailed presentation of the differences in the results of using the two model is given in Appendix 3.2.

The one-year and twenty-year rates are graphed in Appendix 3.3. It is easily seen that they follow each other fairly closely, with the one year rate normally below the twenty year rate. This results in the yield curve having a positive slope over most of the years. The graph of the yield curve over the same time period (1960-1989) is directly below the first graph. The estimated yield curve closely follows the actual yield curve for the same time periods.

## INFLATION

In one of the discussions, Mr. John Mereu comments:

When inflation varies, I would recommend a refinement to the model to provide for a randomly moving point of central tendency that simulates the changing rate of inflation. This will result in a wider spectrum of interest rate tracks to plausible futures. (Mereu, 1988)

There are a number of ways in which to measure inflation. The government calculates two commonly used measures. The first is the Implicit Price Deflator (IPD). This is a Pasche price index, distinguished by the use of the current year's quantities. The formula for calculating a Pasche index is given below:

$$P_p = \frac{\sum (P_1 \times Q_1)}{\sum (P_0 \times Q_1)}$$

where

$P_0$  = Prices in base year

$P_1$  = Prices in current year

$Q_1$  = Quantities purchased in current year

The second measure of inflation involves the use of the Consumer Price Index (CPI). This index is a Laspeyre price index, characterized by the use of the previous years prices. The formula for calculating a Laspeyre index is given below

$$P_L = \frac{\sum (P_1 \times Q_1)}{\sum (P_0 \times Q_1)}$$

where

$P_0$  = Prices in base year

$P_1$  = Prices in current year

$Q_0$  = Quantities purchased in base year

A percentage change in the CPI can be used as a measure of inflation. The Stocks Bonds Bills and Inflation (SBBI) Yearbook for 1989 gives the needed data current to 1988.

The question to be answered is whether inflation really does have an effect on interest rates. Appendix 6.1 gives the raw data for 1960 through 1988. I chose to start at 1960 simply because past data would most likely only cloud the issue. Appendix 6.3 gives a graphical picture of the one-year Treasury rate and the inflation rate as measured by the CPI. The years 1960 to 1965 were characterized by fairly low inflation -- never rising above 2.00%, also with fairly low interest rates. In the following years inflation and the interest rate start a steady increase until 1981 when the inflation rate takes a steep fall one year ahead of a similar fall in interest rates. This fall continues to around 1986 when a small rise begins.

Theoretically, the nominal (observed) interest rate is made up of two components. The first is the real rate of inflation, and the second is the expected rate of inflation.

$$i = r + \pi_e + (r \times \pi_e)$$

where

$i$  = nominal interest rate

$r$  = real interest rate

$\pi_e$  = the expected rate of inflation

Many time the term  $r \times \pi_e$  is omitted because it is always very close to zero. This theory helps to explain the fall in the inflation rate "leading" the fall in interest rates in 1981. As the financial community saw the inflation rate falling for the first time in a decade, they adjusted their expectation downward, thus lowering the nominal interest rate.

A least-squares regression was also performed on this data. The regression yielded the following equation:

$$\text{Interest Rate} = 4.1 + (0.556 * \text{Inflation Rate})$$

The coefficients of the regression are statistically significant, however the  $R^2$  measure was only 42.3%. That is, the inflation rate is able to explain only 42.3% of the variance in the interest rate.

Some interesting observations on this data can be made. The mean interest rate over this period was almost 7 percent (standard deviation = 2.997) while the mean inflation rate was a lower 5% (standard deviation = 3.505). This of course would generate only a 2% real return to the investor on a

one-year Treasury bill. Further, there are a number of instances where there was actually a negative real rate of interest. These occurred in the years 1973, 1974, 1975, 1977, 1978, 1979, and 1980. One should also notice that these are the years characterized by a high rate of inflation.

In summary, we can be certain that there is a definite influence on interest rates from the role of inflation; however, it is difficult to make a numerical measure of this effect. This is an area very open to further research. I have included a great deal of raw data in Appendix 6 for this purpose.

#### STATISTICAL ANALYSIS

The statistical analysis done in this paper was performed using MINITAB -- a statistical package available on Ball State's VAX computer system. After logging on, simply type MINI to enter the program. The data is entered by using the SET command which puts the actual observations into columns. Simply typing DESCRIBE and a column name outputs the number of observations in the column, the mean, median, standard deviation, minimum observation, maximum observation, the first quartile, second quartile, and a few other statistical measures. The package itself provides a whole spectrum of statistical operations. By typing HELP COMMANDS one receives a listing of the available commands.

By typing HELP followed by a specific command, one receives very detailed and easily understood instructions on the use of that command. The help files available from MINITAB are very informative. I direct the interested reader to explore the system by use of the HELP commands as I did. In a matter of a few minutes you will be doing rather tedious calculations with little difficulty and with complete confidence.

The actual programs used to generate the interest rates are printed at the end of this paper. They were written in Turbo PASCAL and an IBM PC. The output from these programs was saved on diskette, and the loaded into ASEASYAS, a spreadsheet program. The graphs, and basic statistical measures on the actual scenarios was performed using ASEASYAS.

#### CONCLUSION

There are a myriad of different ways for insurance companies to deal with interest rate risk. I have concentrated on one specific way, that of generating random interest rates. As has been shown throughout this paper, the methods of actually generating these rates are virtually unlimited. There is a great deal of opinion and judgement which must go into the generation, and in many respects it is more of an art than a science. Which is as it should be, for no one can predict the future with perfect accuracy.

The actual choice of a particular interest rate generator will rest with the actual user of the resulting scenarios. The uses of the rates and the decisions that will be made based on these rates will always be the determining factor in the choice of a scenario generator. The user must therefore be very careful in the assumptions inherent in a particular model, and greatest of all, analyze the results to choose the best model.

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## Appendix 1.1

One-Year Treasury Rates 1960-1989  
 Compiled from: Federal Reserve Bulletin

ROW	1-Yr	lag	Delta I	$\ln((I+dI)/I)$	abs d	
1	3.41	*	*	*	*	
2	2.89	3.41	-0.52	-0.198367	0.52	
3	3.01	2.89	0.12	0.039093	0.12	
4	3.30	3.01	0.29	0.084230	0.29	
5	3.74	3.30	0.44	0.111226	0.44	
6	4.06	3.74	0.32	0.075866	0.32	
7	5.07	4.06	1.01	0.181664	1.01	
8	4.71	5.07	-0.36	-0.079512	0.36	
9	5.46	4.71	0.75	0.128712	0.75	
10	6.79	5.46	1.33	0.178879	1.33	
11	6.49	6.79	-0.30	-0.047327	0.30	
12	4.67	6.49	-1.82	-0.493840	1.82	
13	4.77	4.67	0.10	0.020748	0.10	
14	7.01	4.77	2.24	0.277286	2.24	
15	7.70	7.01	0.69	0.085820	0.69	
16	6.28	7.70	-1.42	-0.256331	1.42	
17	5.88	6.28	-0.40	-0.070452	0.40	
18	6.09	5.88	0.21	0.033902	0.21	
19	8.34	6.09	2.25	0.238847	2.25	
20	10.67	8.34	2.33	0.197513	2.33	
21	12.05	10.67	1.38	0.108426	1.38	
22	14.78	12.05	2.73	0.169497	2.73	
23	12.27	14.78	-2.51	-0.228865	2.51	
24	9.57	12.27	-2.70	-0.331469	2.70	
25	10.89	9.57	1.32	0.114410	1.32	
26	8.43	10.89	-2.46	-0.345050	2.46	
27	6.45	8.43	-1.98	-0.366692	1.98	
28	6.77	6.45	0.32	0.046184	0.32	
29	7.65	6.77	0.88	0.108884	0.88	
30	8.53	7.65	0.88	0.098184	0.88	
1-Yr	N	MEAN	MEDIAN	TRMEAN	STDEV	SEMEAN
	30	6.924	6.470	6.722	2.992	0.546
1-Yr	MIN	MAX	Q1	Q3		
	2.890	14.780	4.700	8.455		
abs d	N	N*	MEAN	MEDIAN	TRMEAN	STDEV
	29	1	1.174	0.880	1.157	0.888
abs d	MIN	MAX	Q1	Q3		
	0.100	2.730	0.340	2.110		
C4	N	N*	MEAN	MEDIAN	TRMEAN	STDEV
	29	1	-0.0041	0.0759	0.0036	0.2022
C4	MIN	MAX	Q1	Q3		
	-0.4938	0.2773	-0.1389	0.1216		

Note:  
 $C4 = \ln((I+dI)/I)$

## Appendix 1.2

Yields on U.S. Government Securities 1960-1989  
Source: Salomon Brothers

ROW	1	lag	Delta I	$\ln((I+dI)/I)$	abs d
1	3.55	*	*	*	*
2	2.89	3.55	-0.66000	-0.259255	0.66000
3	3.05	2.89	0.16000	0.051129	0.16000
4	3.29	3.05	0.24000	0.070410	0.24000
5	3.80	3.29	0.51000	0.125937	0.51000
6	4.07	3.80	0.27000	0.064231	0.27000
7	5.12	4.07	1.05000	0.186544	1.05000
8	4.77	5.12	-0.35000	-0.076207	0.35000
9	5.54	4.77	0.77000	0.130141	0.77000
10	6.95	5.54	1.41000	0.184717	1.41000
11	7.05	6.95	0.10000	0.014085	0.10000
12	4.89	7.05	-2.16000	-0.582891	2.16000
13	4.88	4.89	-0.01000	-0.002051	0.01000
14	7.24	4.88	2.36000	0.282142	2.36000
15	8.23	7.24	0.99000	0.113589	0.99000
16	6.65	8.23	-1.58000	-0.271276	1.58000
17	5.92	6.65	-0.73000	-0.131603	0.73000
18	5.94	5.92	0.02000	0.003361	0.02000
19	8.20	5.94	2.26000	0.243424	2.26000
20	10.54	8.20	2.34000	0.200498	2.34000
21	12.07	10.54	1.53000	0.119347	1.53000
22	14.73	12.07	2.66000	0.166009	2.66000
23	12.40	14.73	-2.33000	-0.208136	2.33000
24	9.47	12.40	-2.93000	-0.370192	2.93000
25	10.92	9.47	1.45000	0.124678	1.45000
26	7.60	10.92	-3.32000	-0.574195	3.32000
27	6.40	7.60	-1.20000	-0.207639	1.20000
28	6.60	6.40	0.20000	0.029853	0.20000
29	7.53	6.60	0.93000	0.116454	0.93000
30	8.60	7.53	1.07000	0.117266	1.07000

	N	MEAN	MEDIAN	TRMEAN	STDEV	SEMEAN
1	30	6.963	6.625	6.762	2.961	0.541

	MIN	MAX	Q1	Q3
1	2.890	14.730	4.852	8.323

	N	N*	MEAN	MEDIAN	TRMEAN	STDEV	SEMEAN
abs d	29	1	1.227	1.050	1.195	0.961	0.178

	MIN	MAX	Q1	Q3
abs d	0.010	3.320	0.310	2.210

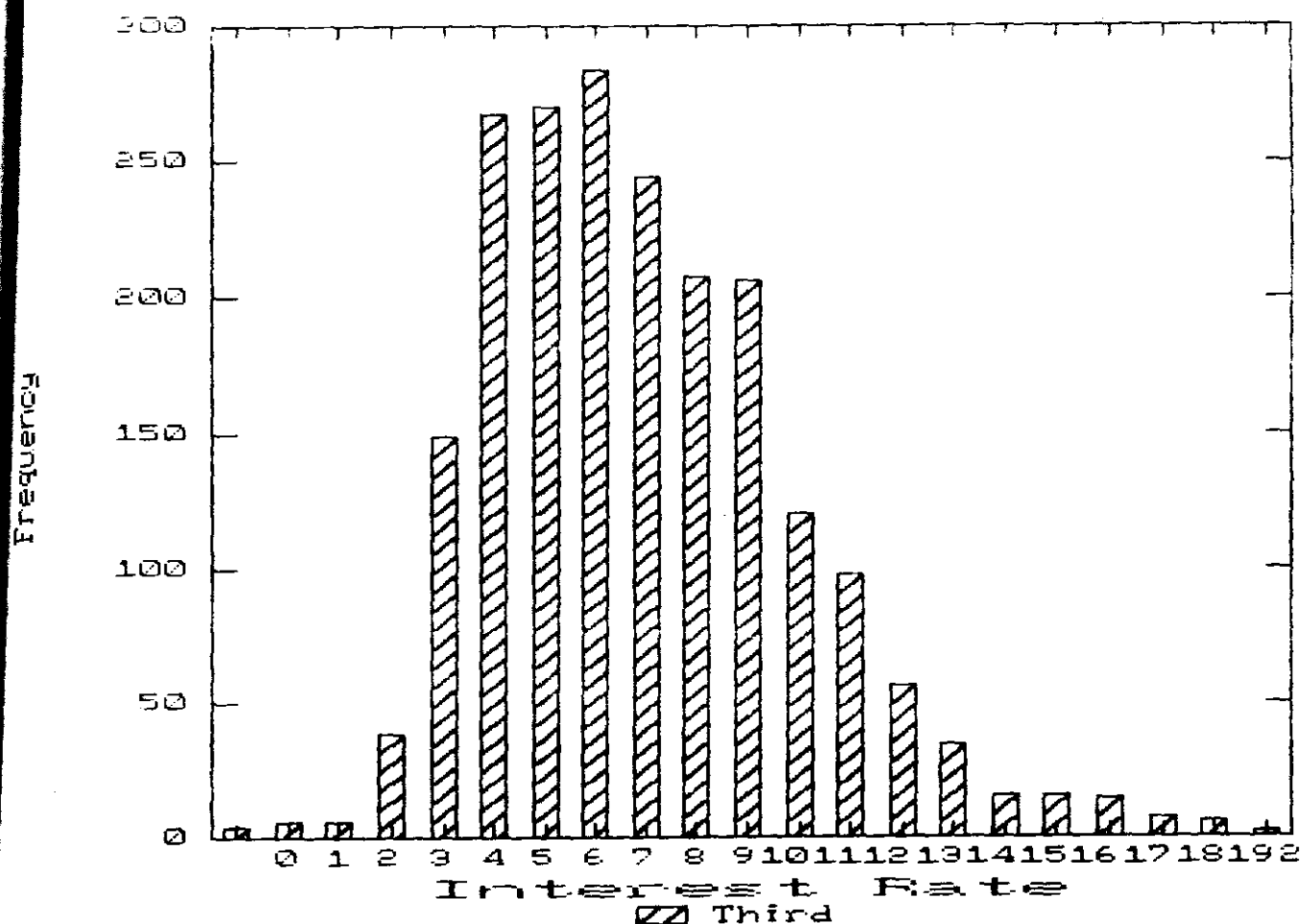
	N	N*	MEAN	MEDIAN	TRMEAN	STDEV	SEMEAN
C9	29	1	-0.0117	0.0642	-0.0014	0.2271	0.0422

	MIN	MAX	Q1	Q3
C9	-0.5829	0.2821	-0.1696	0.1280

Note:  
C9= $\ln((I+dI)/I)$

# Third Stochastic Model



Model: Third Stochastic

Rates: One-Year Treasury  
 Scenarios: 100  
 Initial Rate: 8.00%  
 Mean-Reverting: Jetton's Original  
 0.015(d<sup>3</sup>) or 0.5d  
 Volatility Factor: 0.27

\$1 accumulated 20 yrs.:

mean 5.0081  
 st. dev. 1.4922

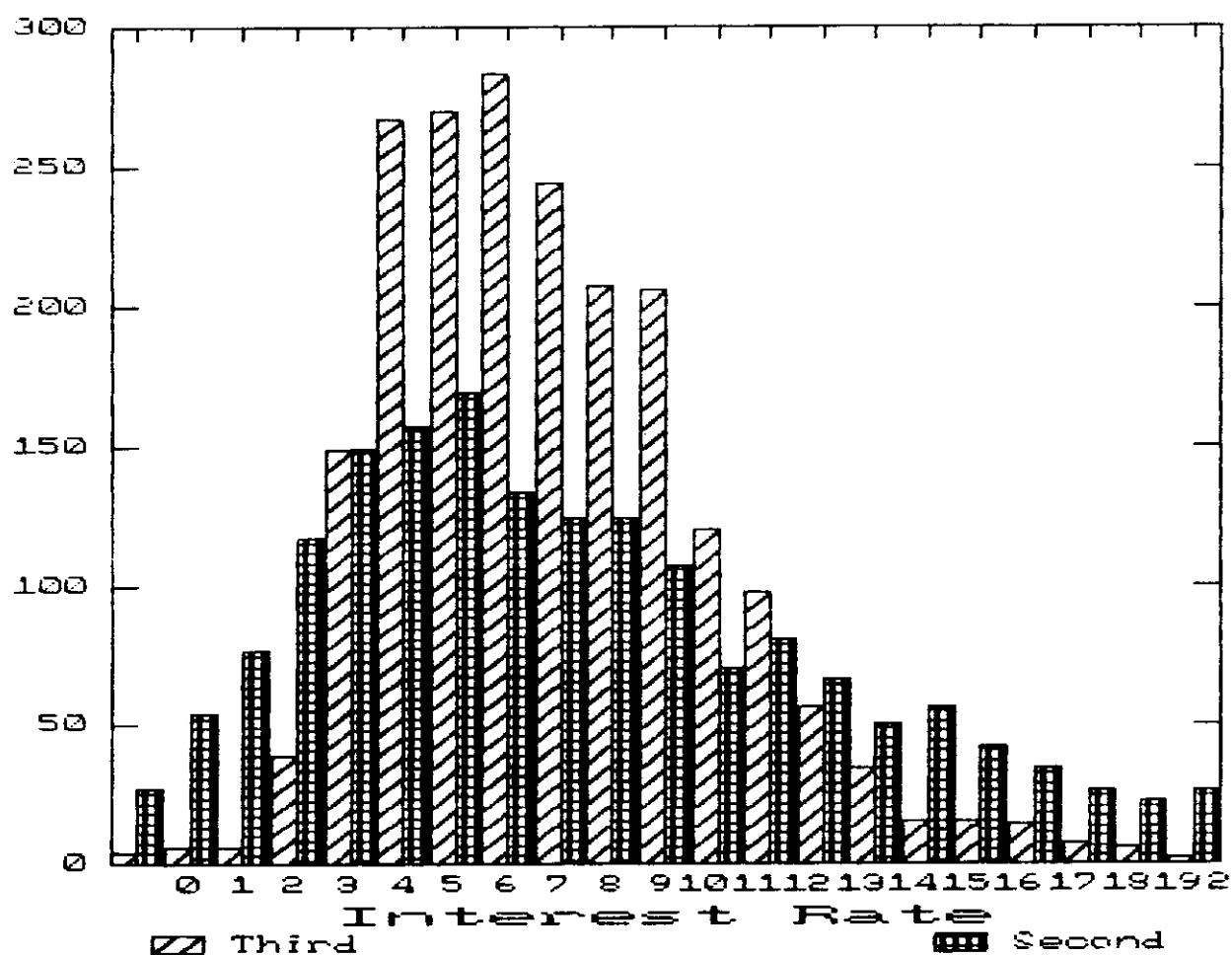
\$1 annuity-due 20-yrs:

mean 51.8093  
 st. dev. 11.8140

Implied constant rate:

mean 8.1718  
 st. dev. 1.7989

## Second Stochastic Model



Model: Second Stochastic

Rates: One-Year Treasury

Scenarios: 100

Initial Rate: 8.00%

Mean-Reverting: Unbounded

Volatility Factor: 0.23

\$1 accumulated 20 yrs.:

mean 104.4033

st. dev. 533.5684

\$1 annuity-due 20-yr:

mean 652.9698

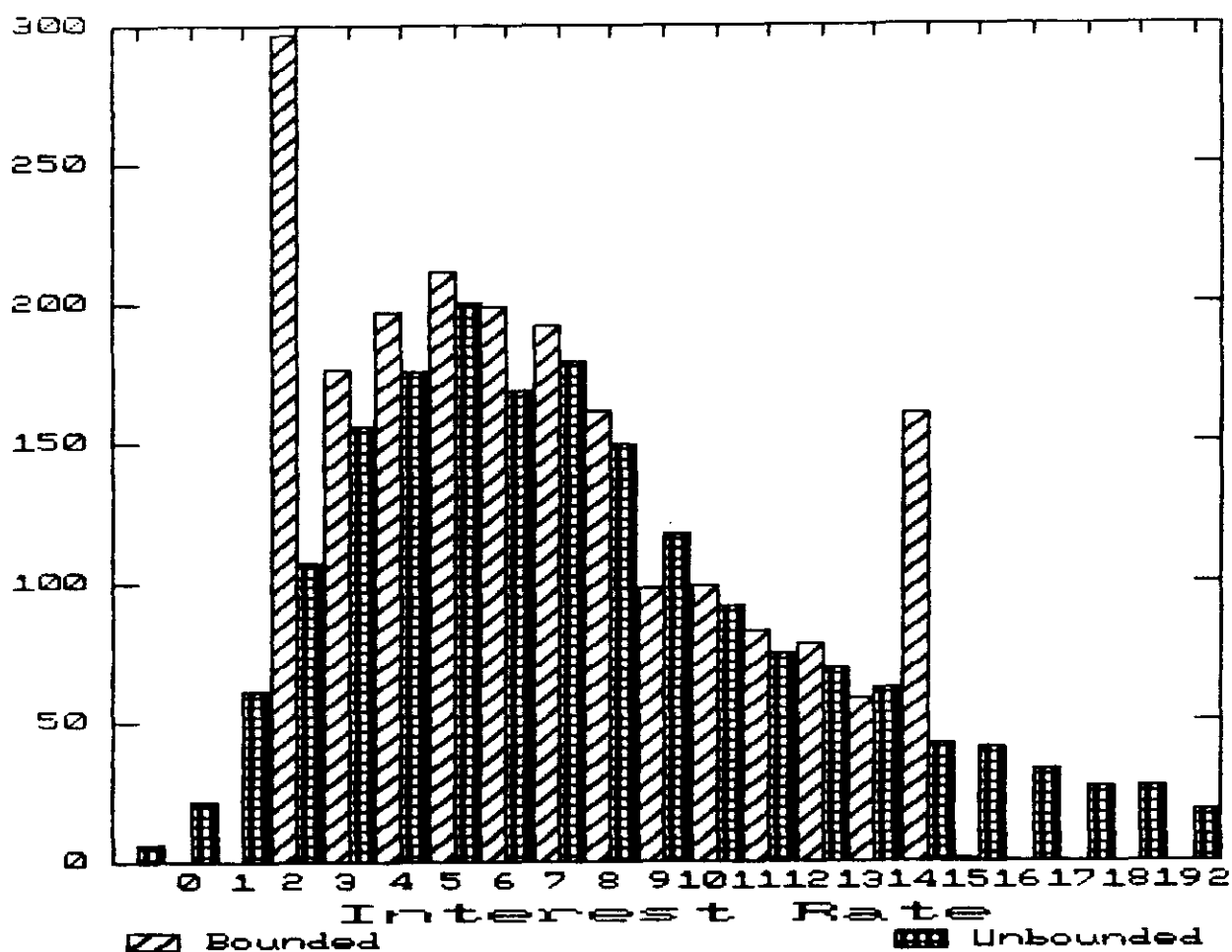
st. dev. 3065.3800

Implied constant rate:

mean 12.0461

st. dev. 8.3566

## Second Stochastic Model



Model: Second Stochastic

Rates: One-Year Treasury

Scenarios: 100

Initial Rate: 8.00%

Mean-Reverting: Bounded

Volatility Factor: 0.23

\$1 accumulated 20 yrs.:

mean	5.2700
st. dev.	2.5295

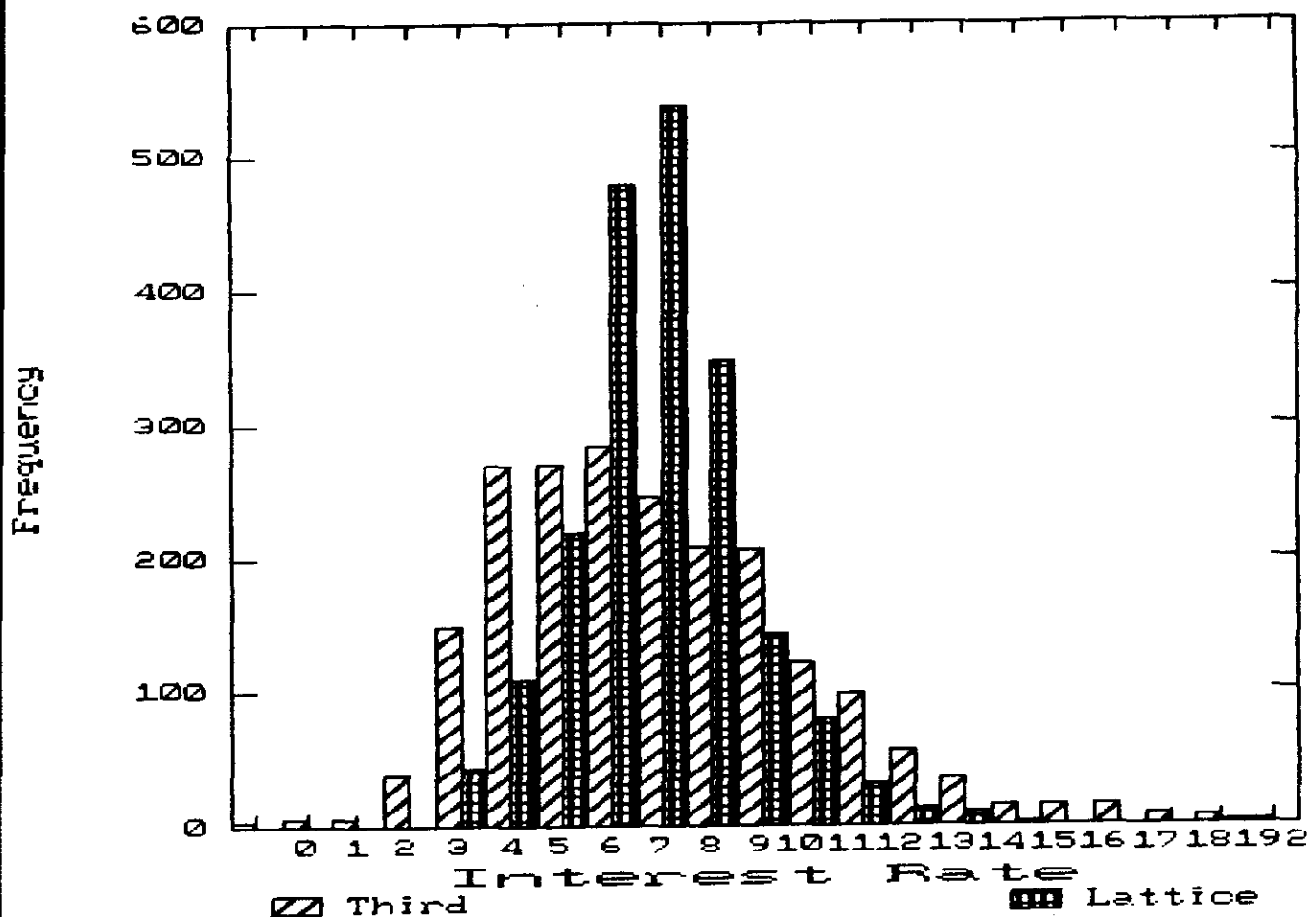
\$1 annuity-due 20-yrs:

mean	52.9194
st. dev.	18.7463

Implied constant rate:

mean	8.0214
st. dev.	2.8665

## Binomial Lattice



Model: Binomial Lattice

Rates: One-Year Treasury

Scenarios: 100

Initial Rate: 8.00%

Mean-Reverting: None

Volatility Factor: Step 0.5

\$1 accumulated 20 yrs.:

mean 4.8216  
st. dev. 1.2800

\$1 annuity-due 20-yrs:

mean 50.3802  
st. dev. 10.2615

Implied constant rate:

mean 7.9685  
st. dev. 1.7124

Appendix 3.1  
Data

Yields on U.S. Government Securities 1960-1989

Source: Salomon Brothers

Regression on slope of the yield curve

ROW	1	20	y=slope	lag 1	x1=lag s	x2 lag
1	3.55	4.13	0.163380	*	*	*
2	2.89	3.90	0.349481	3.55	0.163380	-0.185915
3	3.05	4.02	0.318033	2.89	0.349481	0.055363
4	3.29	4.04	0.227964	3.05	0.318033	0.078688
5	3.80	4.18	0.100000	3.29	0.227964	0.155015
6	4.07	4.23	0.039312	3.80	0.100000	0.071053
7	5.12	4.72	-0.078125	4.07	0.039312	0.257985
8	4.77	4.93	0.033543	5.12	-0.078125	-0.068359
9	5.54	5.40	-0.025271	4.77	0.033543	0.161426
10	6.95	6.25	-0.100719	5.54	-0.025271	0.254513
11	7.05	6.79	-0.036879	6.95	-0.100719	0.014389
12	4.89	6.01	0.229039	7.05	-0.036879	-0.306383
13	4.88	5.80	0.188525	4.89	0.229039	-0.002045
14	7.24	6.97	-0.037293	4.88	0.188525	0.483606
15	8.23	7.93	-0.036452	7.24	-0.037293	0.136740
16	6.65	8.04	0.209023	8.23	-0.036452	-0.191980
17	5.92	7.86	0.327703	6.65	0.209023	-0.109774
18	5.94	7.62	0.282828	5.92	0.327703	0.003378
19	8.20	8.42	0.026829	5.94	0.282828	0.380471
20	10.54	9.24	-0.123340	8.20	0.026829	0.285366
21	12.07	11.29	-0.064623	10.54	-0.123340	0.145161
22	14.73	13.69	-0.070604	12.07	-0.064623	0.220381
23	12.40	13.07	0.054032	14.73	-0.070604	-0.158181
24	9.47	11.27	0.190074	12.40	0.054032	-0.236290
25	10.92	12.49	0.143773	9.47	0.190074	0.153115
26	7.60	10.60	0.394737	10.92	0.143773	-0.304029
27	6.40	7.80	0.218750	7.60	0.394737	-0.157895
28	6.60	8.70	0.318182	6.40	0.218750	0.031250
29	7.53	9.00	0.195219	6.60	0.318182	0.140909
30	8.60	8.58	-0.002326	7.53	0.195219	0.142098

$$Y = c + aX_1 + bX_2$$

where  $Y = \text{slope} = S(t) = [T_{20}(t) - T_1(t)]/T_1(t)$

$X_1 = S(t)$

$X_2 = [T_1(t+1) - T_1(t)]/T_1(t)$

Appendix 3.2  
Results

Yields on U.S. Government Securities  
Source: Salomon Brothers  
Regression on slope of the yield curve

$$y = 0.0669 + 0.627 x_1 - 0.569 x_2$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.06693	0.01519	4.41	0.000
x1	0.62691	0.07786	8.05	0.000
x2	-0.56857	0.06055	-9.39	0.000

s = 0.06381      R-sq = 84.6%      R-sq(adj) = 83.4%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	0.58090	0.29045	71.34	0.000
Error	26	0.10585	0.00407		
Total	28	0.68675			

SOURCE	DF	SEQ SS
x1	1	0.22189
x2	1	0.35901

Unusual Observations

Obs.	x1	y	Fit	Stdev.Fit	Residual	St.Resid
26	0.395	0.2188	0.4042	0.0283	-0.1854	-3.24R
27	0.219	0.3182	0.1863	0.0143	0.1319	2.12R

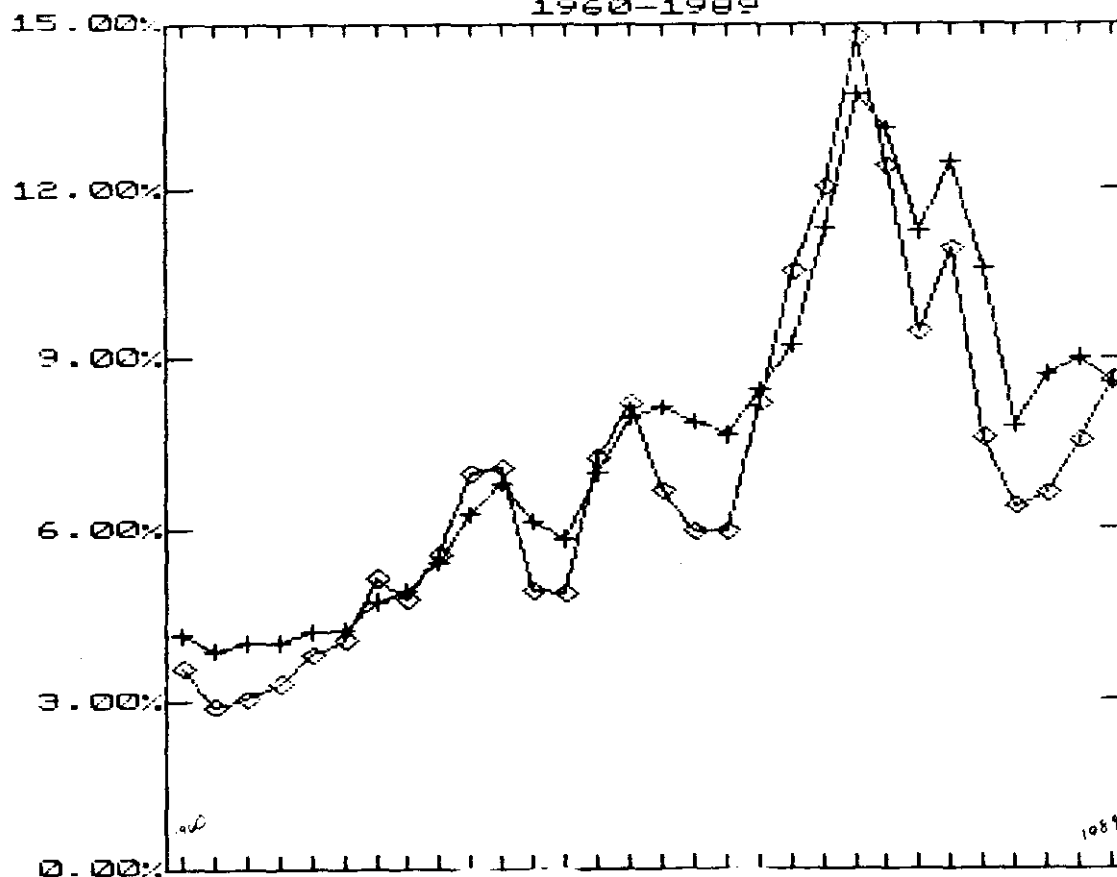
R denotes an obs. with a large st. resid.



## Historical Interest Rates

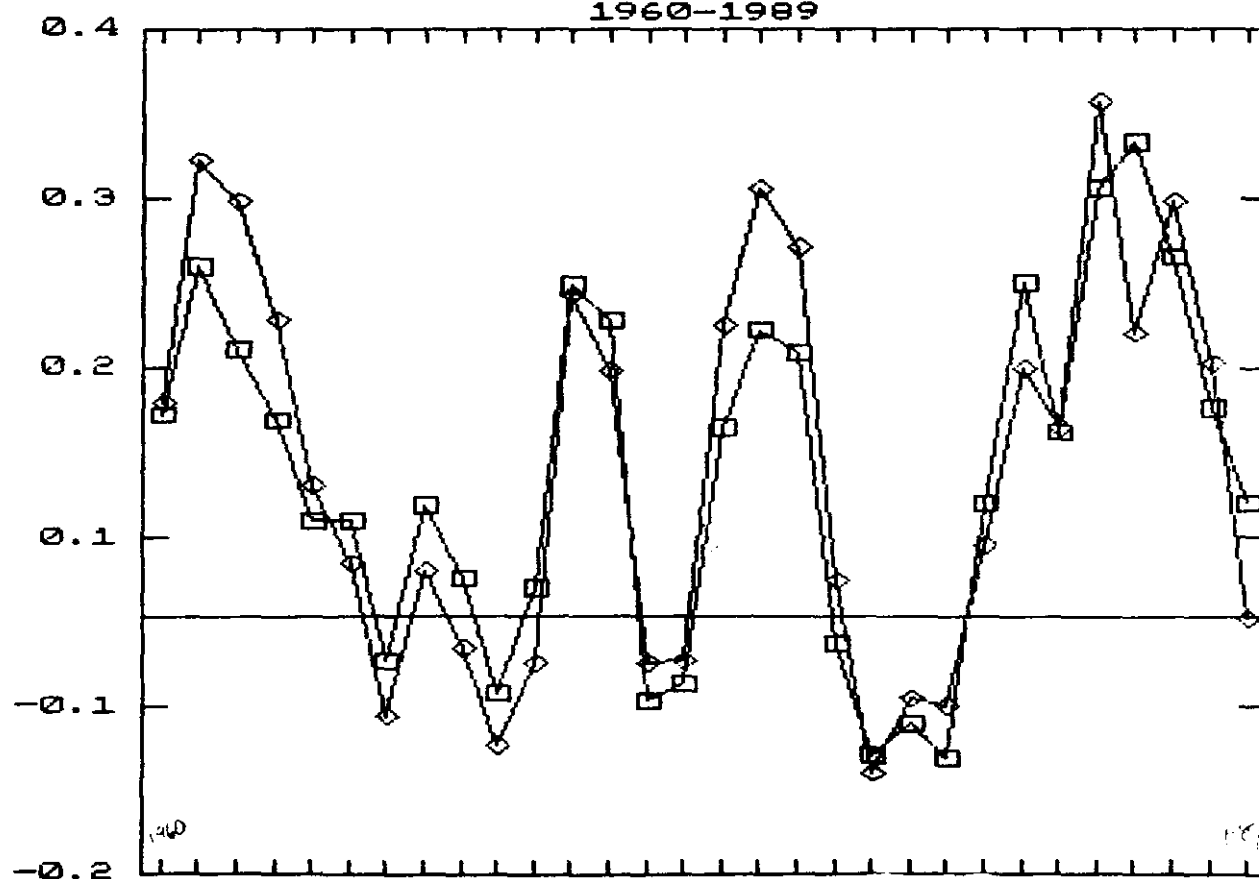
1960-1989

Rate

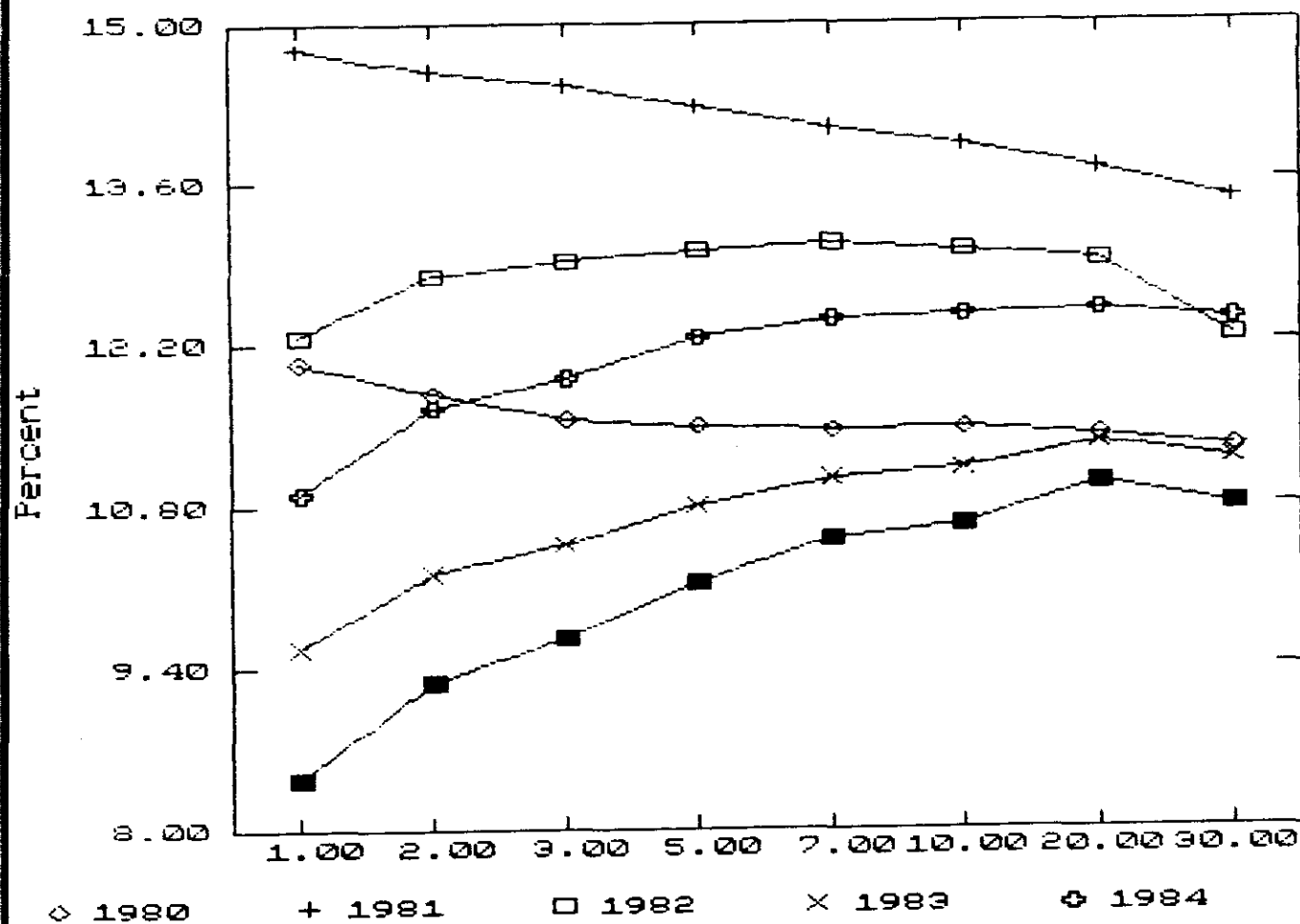


## Yield Curve Slope

1960-1989

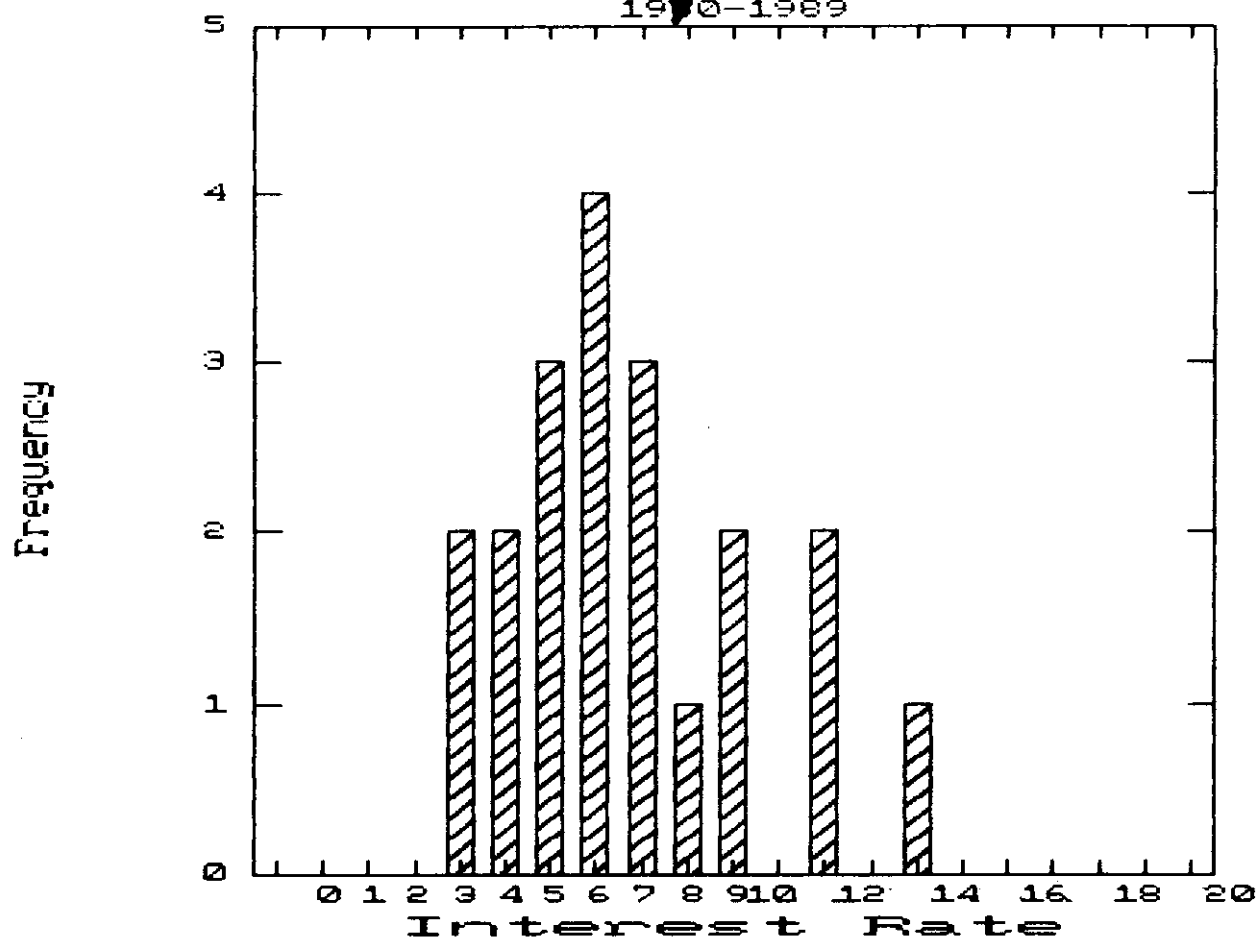
Slope  $S(t)$ 

## Yield Curve Slope



Year	Bonds						Notes	
	1	2	3	5	7	10	20	30
1980	12.05	11.77	11.55	11.48	11.43	11.46	11.39	11.30
1981	14.78	14.56	14.44	14.24	14.06	13.91	13.71	13.44
1982	12.27	12.80	12.92	13.01	13.06	13.00	12.92	12.26
1983	9.57	10.21	10.45	10.80	11.02	11.10	11.34	11.18
1984	10.89	11.65	11.89	12.24	12.40	12.44	12.48	12.39
1985	8.43	9.27	9.64	10.12	10.50	10.62	10.97	10.79

# Historical Interest Rates 1970-1989



Model: Historical Rates

Rates: One-Year Treasury

Scenarios: 1

Initial Rate: N/A

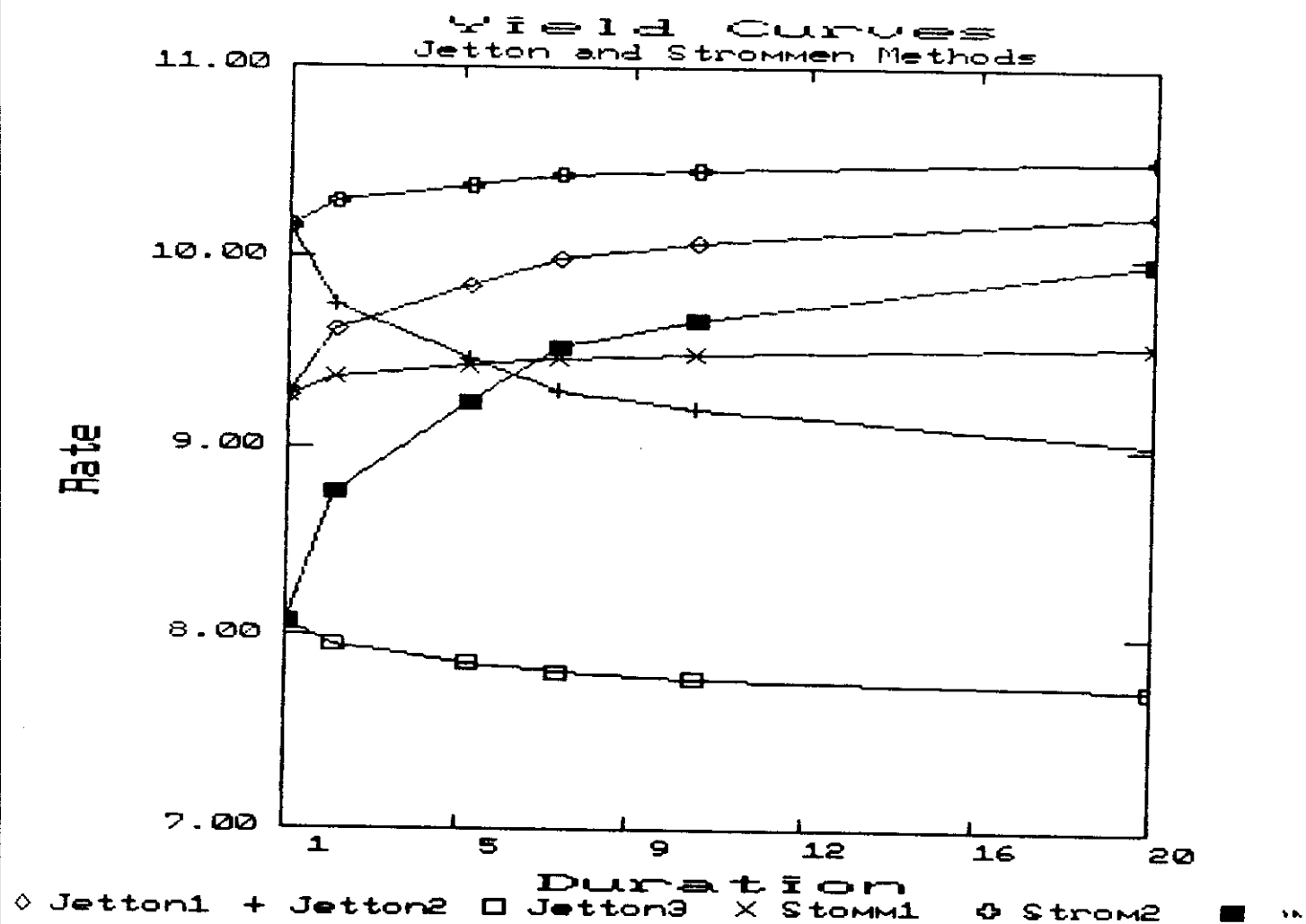
Mean-Reverting: N/A

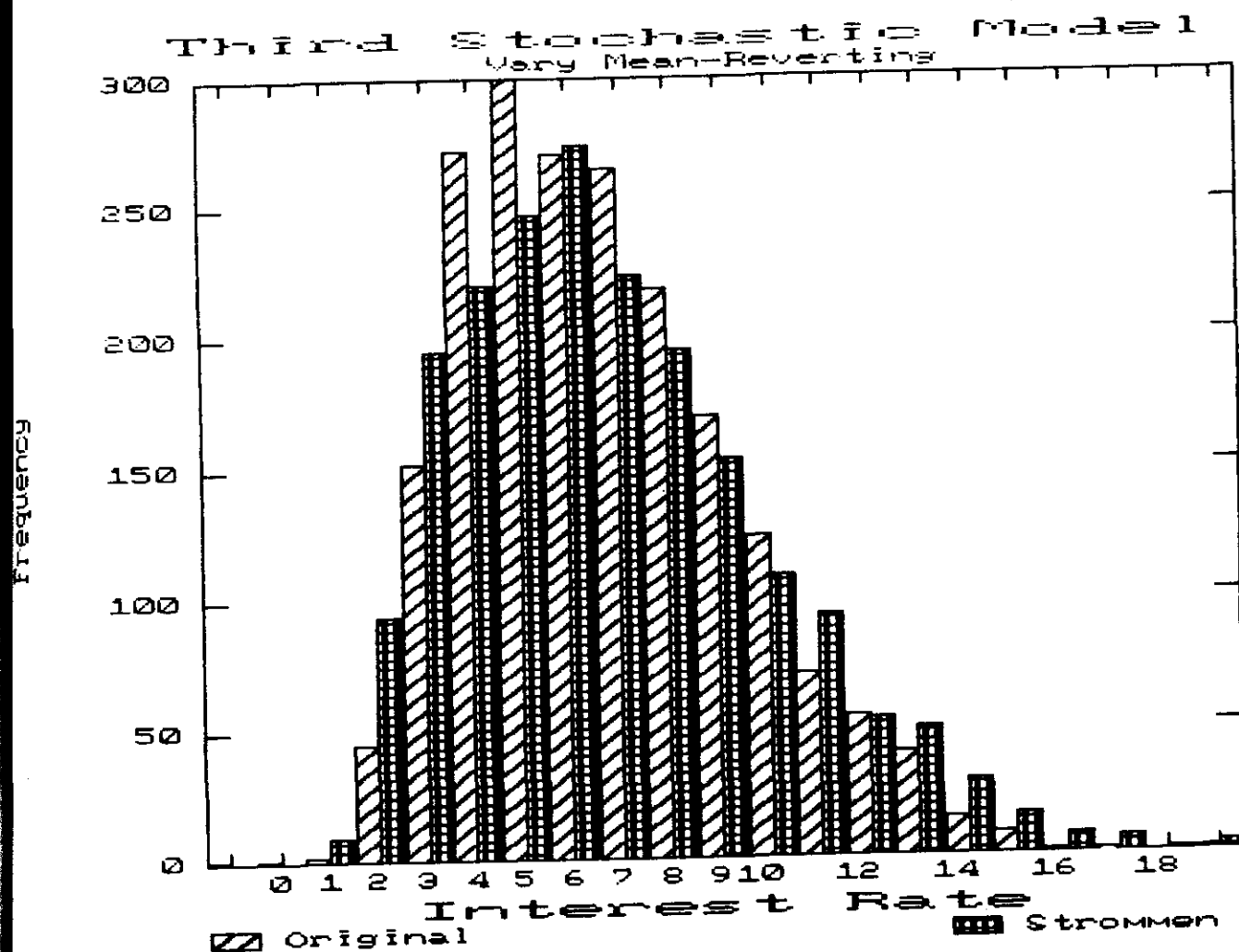
Volatility Factor: N/A

\$1 accumulated 20 yrs.: 4.893

\$1 annuity-due 20-yrs: 53.949

Implied constant rate: 8.73





Model: Third Stochastic

Rates: One-Year Treasury

Scenarios: 100

Initial Rate: 8.00%

Mean-Reverting: Strommen's Range

Volatility Factor: 0.27

\$1 accumulated 20 yrs.:

mean 5.0546  
st. dev. 1.6044

\$1 annuity-due 20-yrs:

mean 51.5685  
st. dev. 11.8184

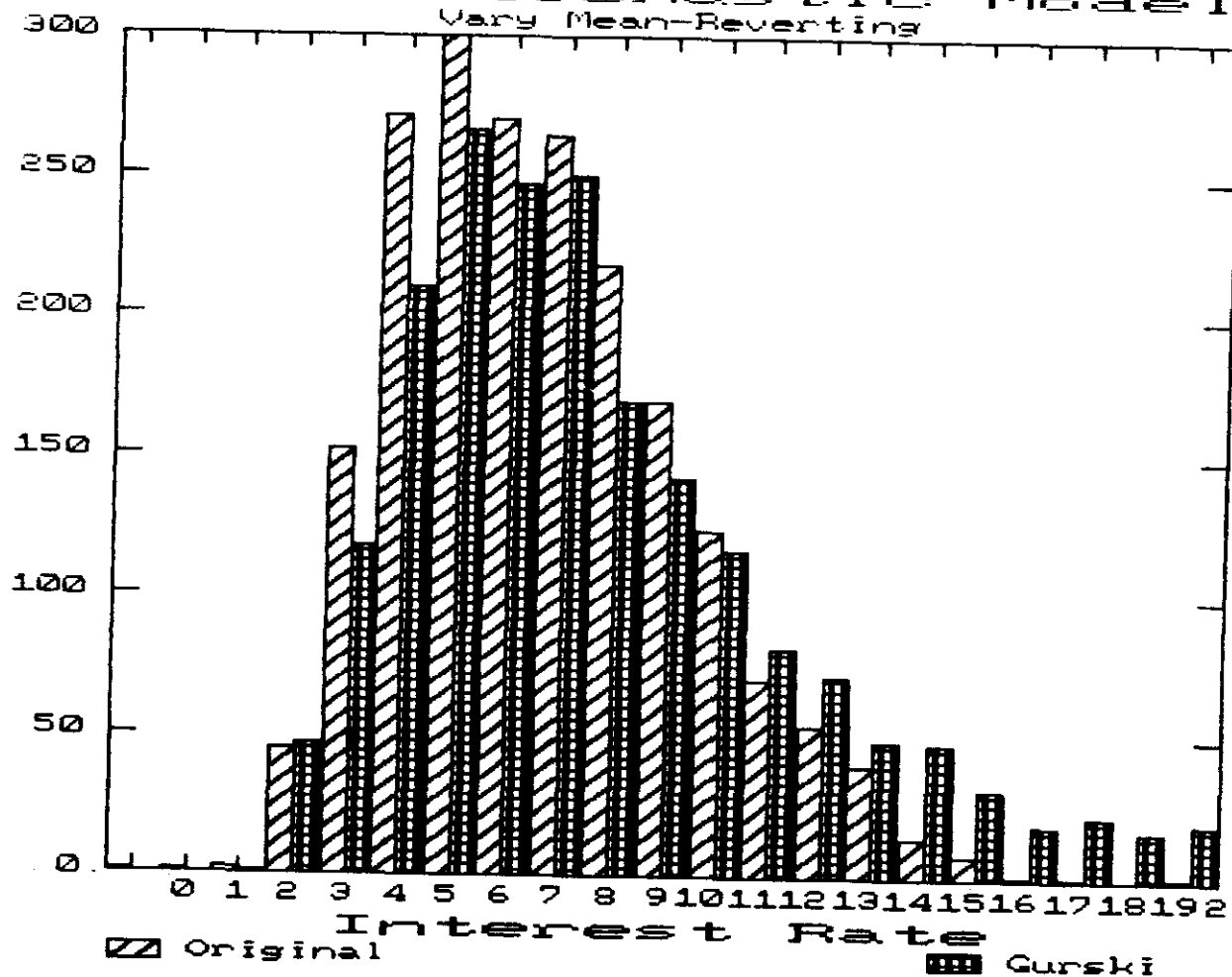
Implied constant rate:

mean 8.1198  
st. dev. 1.8757

# Third Stochastic Model

## Vary Mean-Reverting

Frequency



Model: Third Stochastic

Rates: One-Year Treasury

Scenarios: 100

Initial Rate: 8.00%

Mean-Reverting: Gurski's

Volatility Factor: 0.27

\$1 accumulated 20 yrs.:

mean 8.7944

st. dev. 10.7354

\$1 annuity-due 20-yrs:

mean 143.8984

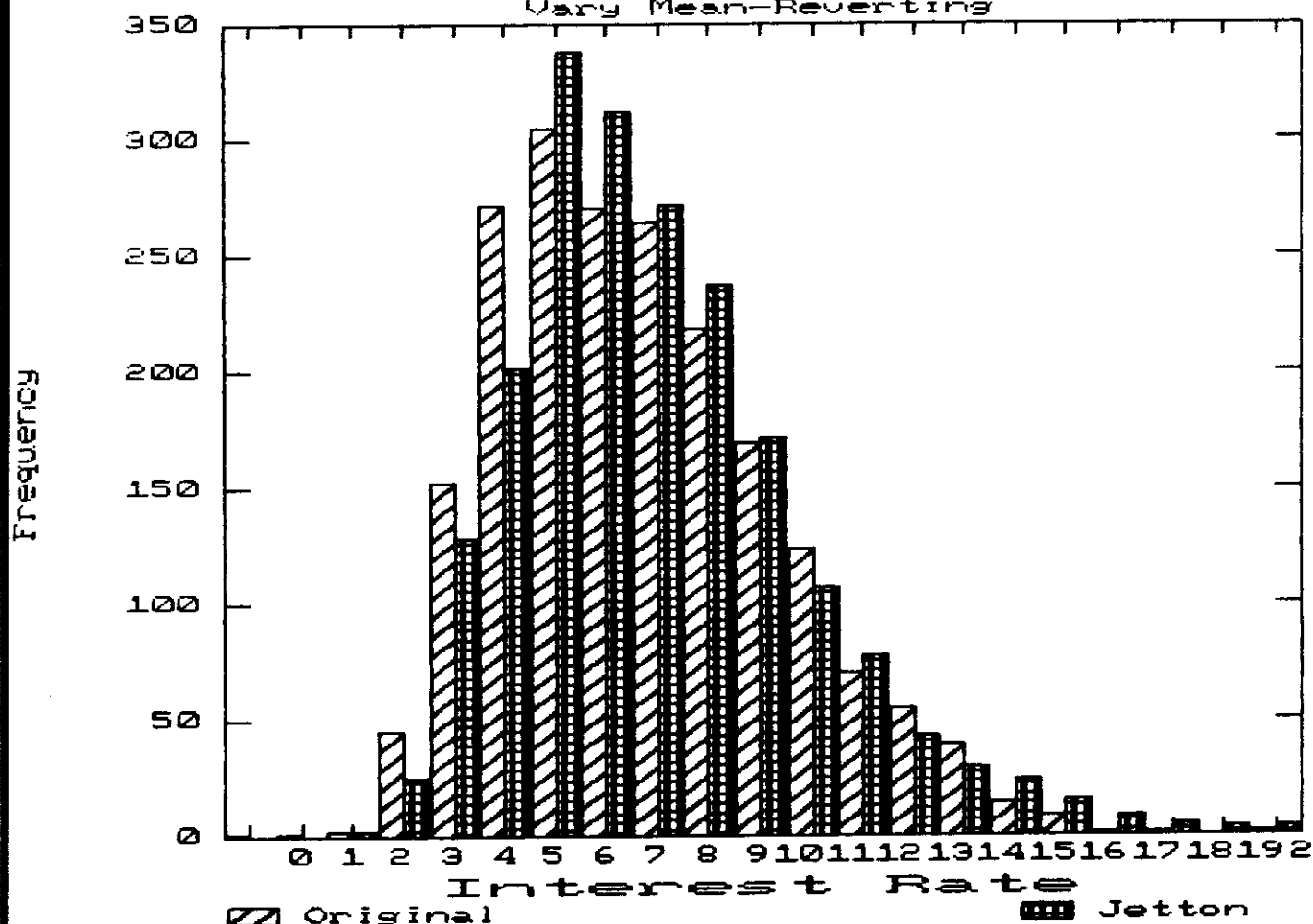
st. dev. 505.6847

Implied constant rate:

mean 10.4679

st. dev. 5.0002

# Third Stochastic Model Vary Mean-Reverting



Model: Third Stochastic

Rates: One-Year Treasury  
 Scenarios: 100  
 Initial Rate: 8.00%  
 Mean-Reverting: Jetton's Original  
 $0.015(d^3)$  or  $0.5d$   
 Volatility Factor: 0.27

\$1 accumulated 20 yrs.:  
 mean 4.9853  
 st. dev. 1.5071

\$1 annuity-due 20-yrs:  
 mean 50.7546  
 st. dev. 11.3224

Implied constant rate:  
 mean 8.1324  
 st. dev. 1.5972

Model: Third Stochastic

Rates: One-Year Treasury  
 Scenarios: 100  
 Initial Rate: 8.00%  
 Mean-Reverting: Jetton's Mod.  
 $0.1d + 0.008d^3$   
 Volatility Factor: 0.27

\$1 accumulated 20 yrs.:  
 mean 5.2380  
 st. dev. 1.4905

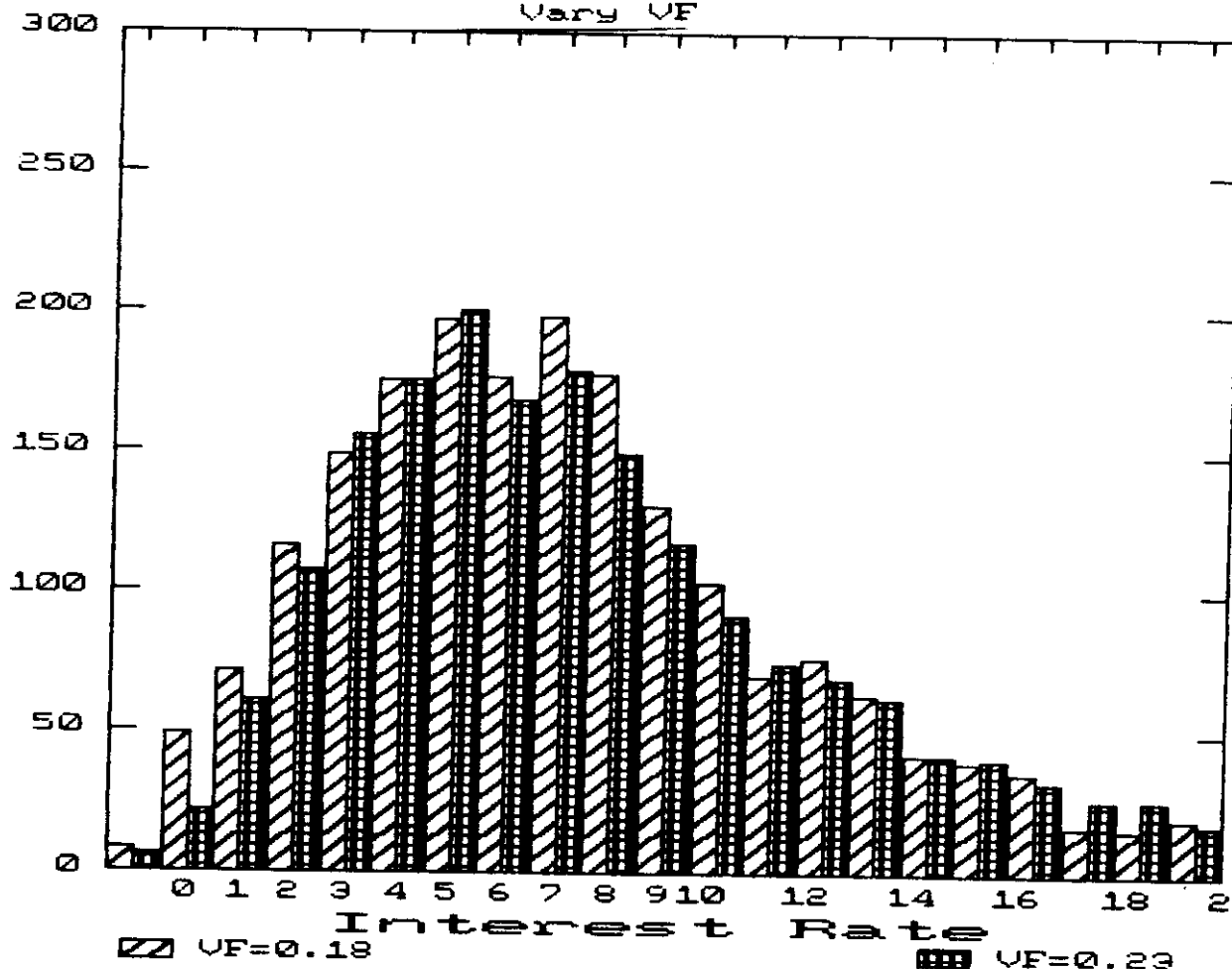
\$1 annuity-due 20-yrs:  
 mean 53.3305  
 st. dev. 12.5271

Implied constant rate:  
 mean 8.5313  
 st. dev. 1.6503

# Second Stochastic Model

Vary VF

Frequency



Model: Second Stochastic

Rates: One-Year Treasury

Scenarios: 100

Initial Rate: 8.00%

Mean-Reverting: Unbounded

Volatility Factor: 0.18

\$1 accumulated 20 yrs.:

mean 11.7037  
st. dev. 29.3464

\$1 annuity-due 20-yrs:

mean 101.4758  
st. dev. 212.8650

Implied constant rate:

mean 10.4889  
st. dev. 6.2877

Model: Second Stochastic

Rates: One-Year Treasury

Scenarios: 100

Initial Rate: 8.00%

Mean-Reverting: Unbounded

Volatility Factor: 0.23

\$1 accumulated 20 yrs.:

mean 32.0714  
st. dev. 179.7972

\$1 annuity-due 20-yrs:

mean 228.2351  
st. dev. 1092.5049

Implied constant rate:

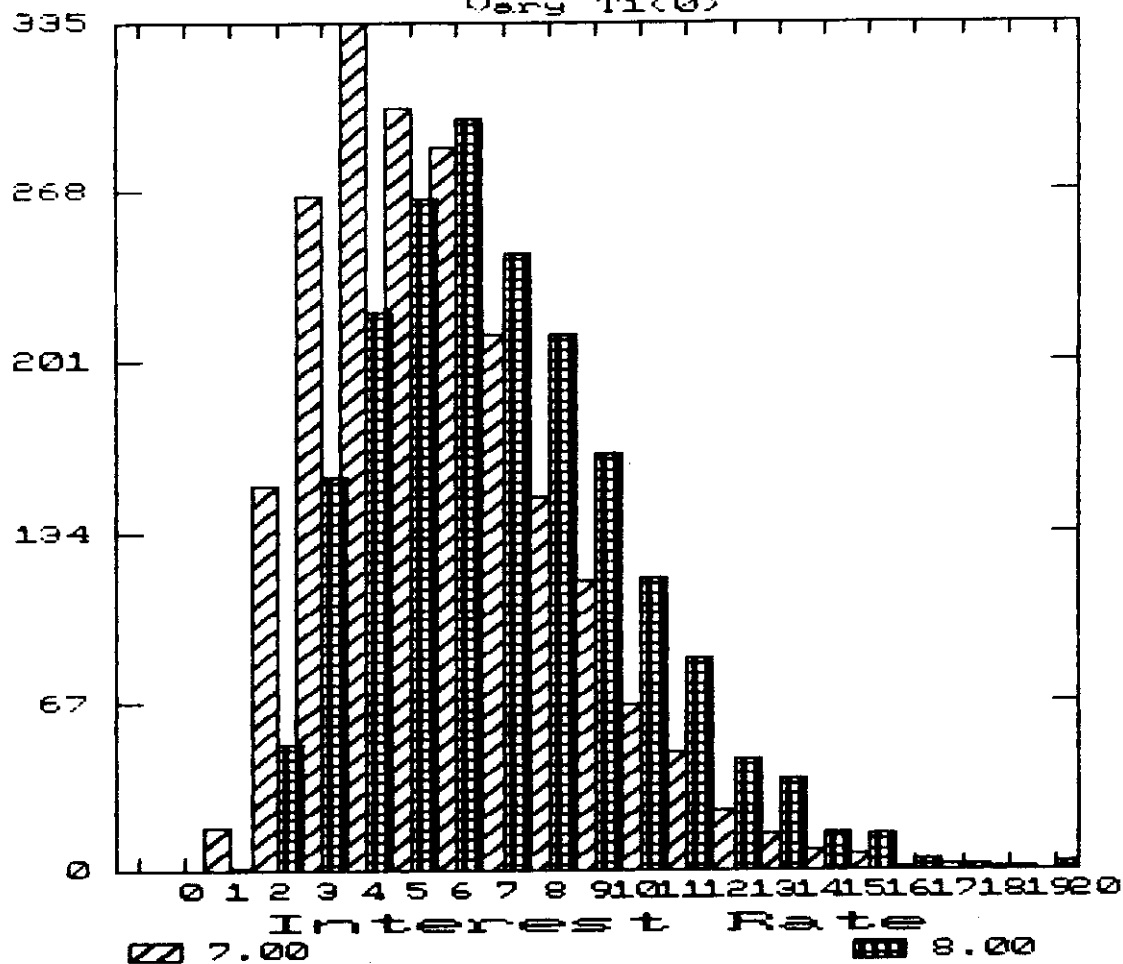
mean 11.4221  
st. dev. 6.0907

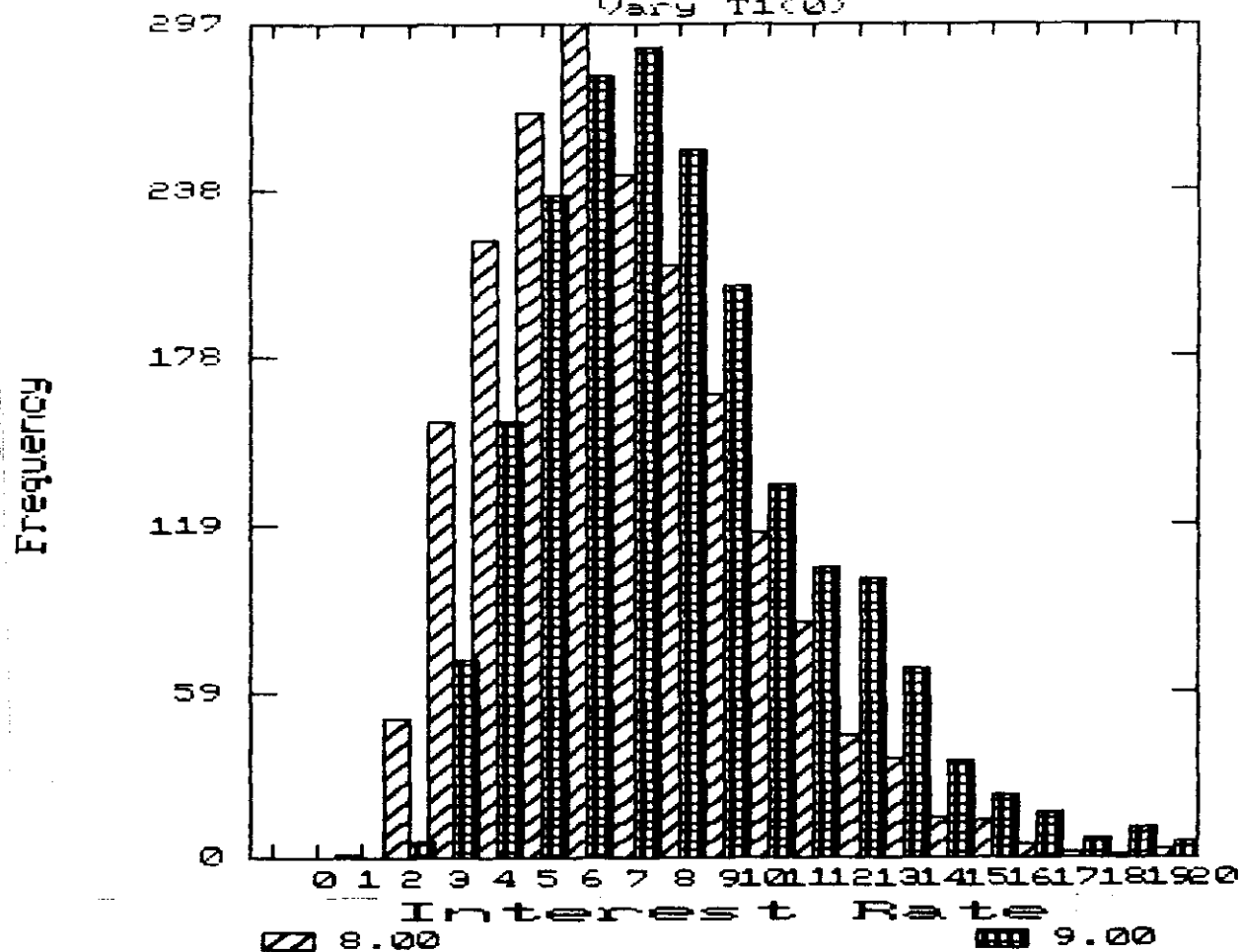


# Third Stochastic Model

Very T1(0)

Frequency



Third Stochastic Model  
Vary T1(0)

One-Year U.S. Gov't Security Yields and  
Consumer Price Index Inflation Rates  
1960-1988

Year	Rate	Inflation
1	3.55	1.48
2	2.89	0.67
3	3.05	1.22
4	3.29	1.65
5	3.80	1.19
6	4.07	1.92
7	5.12	3.35
8	4.77	3.04
9	5.54	4.72
10	6.95	6.11
11	7.05	5.49
12	4.89	3.36
13	4.88	3.41
14	7.24	8.80
15	8.23	12.20
16	6.65	7.01
17	5.92	4.81
18	5.94	6.77
19	8.20	9.03
20	10.54	13.31
21	12.07	12.40
22	14.73	8.94
23	12.40	3.87
24	9.47	3.80
25	10.92	3.95
26	7.60	3.77
27	6.40	1.13
28	6.60	4.41
29	7.53	4.42

Sources: Yields from Saloman Brothers  
Inflation Rates from SBBI 1989 Yearbook p.166

## Appendix 6.2

The regression equation is  
 $\text{Rate} = 4.10 + 0.556 \text{ Inflation}$

Predictor	Coef	Stdev	t-ratio	p
Constant	4.1029	0.7634	5.37	0.000
Inflation	0.5560	0.1250	4.45	0.000

s = 2.319

R-sq = 42.3%

R-sq(adj) = 40.1%

## Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	106.34	106.34	19.78	0.000
Error	27	145.18	5.38		
Total	28	251.52			

## Unusual Observations

Obs.	Inflat.	Rate	Fit	Stdev.Fit	Residual	St.Resid
20	13.3	10.540	11.503	1.120	-0.963	-0.47 X
22	8.9	14.730	9.074	0.650	5.656	2.54R
23	3.9	12.400	6.255	0.455	6.145	2.70R
25	4.0	10.920	6.299	0.452	4.621	2.03R

R denotes an obs. with a large st. resid.

X denotes an obs. whose X value gives it large influence.

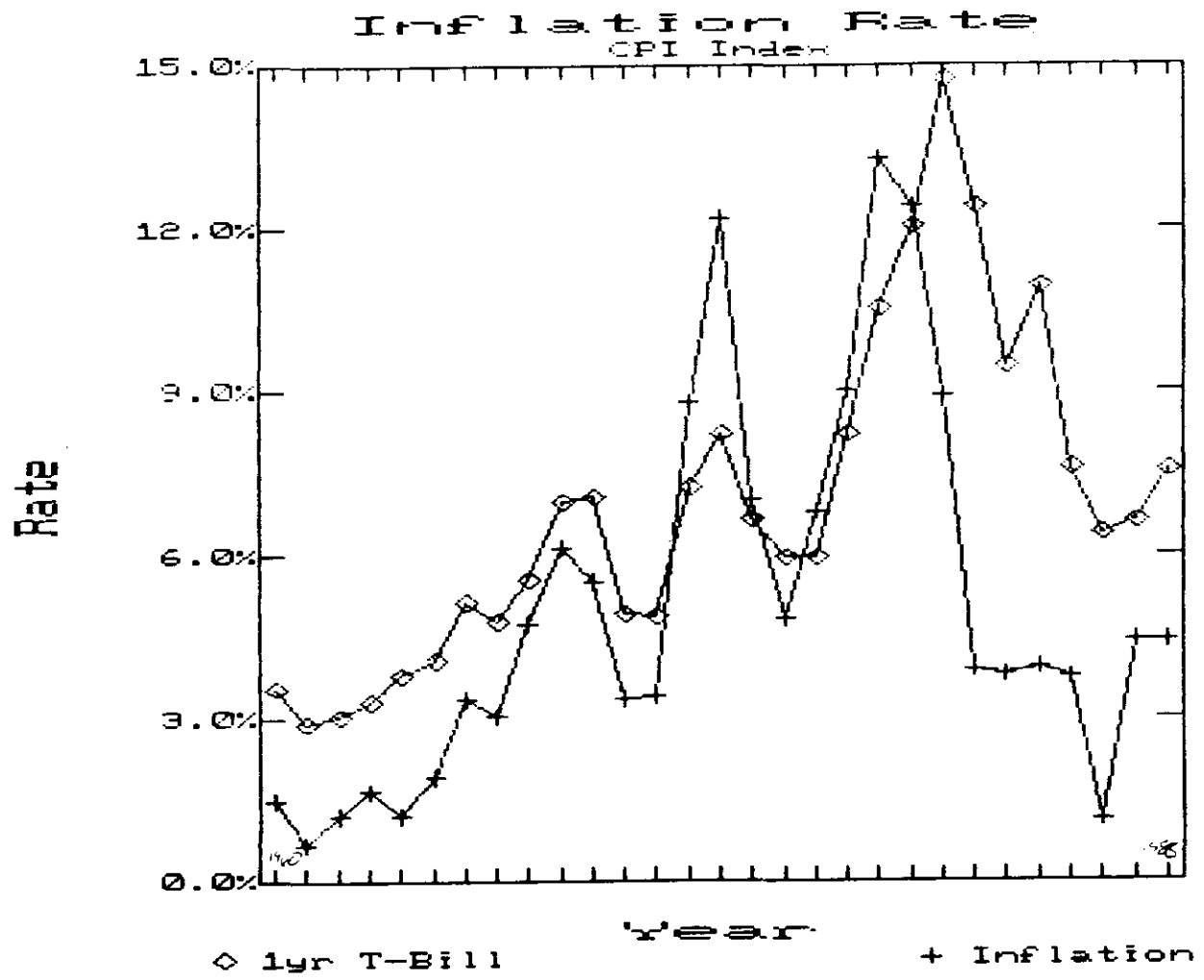
Correlation of Rate and Inflation = 0.650

## VARIANCE-COVARIANCE MATRIX

	Rate	Inflation
Rate	8.98299	
Inflation	6.83088	12.28558

	N	MEAN	MEDIAN	TRMEAN	STDEV	SEMEAN
Rate	29	6.907	6.600	6.766	2.997	0.557
Inflation	29	5.042	3.950	4.898	3.505	0.651

	MIN	MAX	Q1	Q3
Rate	2.890	14.730	4.825	8.215
Inflation	0.670	13.310	2.480	6.890



# IMATRIX

100	3MO	6MO	1YR	2YR	3YR	5YR	7YR	10YR	20YR	30YR
1	1.19	1.24	1.40	1.50	1.74	2.10	2.55	2.55	2.72	2.92
2	0.94	1.00	1.28	1.32	1.53	1.99	2.49	2.49	2.64	2.91
3	1.02	1.07	1.48	1.32	1.48	1.83	2.40	2.40	2.58	2.76
4	1.01	1.05	1.90	1.29	1.45	1.75	2.32	2.32	2.52	2.76
5	0.77	0.81	1.75	1.15	1.38	1.82	2.38	2.38	2.58	2.75
6	0.63	0.68	1.80	1.05	1.34	1.76	2.38	2.38	2.57	2.74
7	0.73	0.74	1.96	0.97	1.23	1.75	2.34	2.34	2.50	2.67
8	0.88	0.90	2.20	1.13	1.37	1.82	2.37	2.37	2.50	2.64
9	1.02	1.05	2.30	1.25	1.45	1.88	2.48	2.48	2.50	2.72
10	0.99	0.99	2.36	1.19	1.45	1.89	2.42	2.42	2.54	2.70
11	0.88	0.91	2.48	1.24	1.50	1.98	2.52	2.52	2.58	2.75
12	1.23	1.22	2.69	1.41	1.65	2.08	2.50	2.50	2.57	2.74
1	1.34	1.36	2.56	1.75	2.04	2.27	2.56	2.56	2.69	2.81
2	1.13	1.18	2.48	1.87	2.12	2.33	2.55	2.55	2.74	2.98
3	1.26	1.33	2.57	2.01	2.15	2.34	2.56	2.56	2.73	2.97
4	1.61	1.71	2.96	2.17	2.28	2.47	2.65	2.65	2.79	2.97
5	1.39	1.51	2.87	2.14	2.30	2.48	2.63	2.63	2.79	2.96
6	1.47	1.58	2.70	2.16	2.34	2.51	2.65	2.65	2.77	2.90
7	1.55	1.69	2.57	2.30	2.46	2.72	2.78	2.78	2.86	2.95
8	1.84	1.96	3.00	2.46	2.62	2.75	2.89	2.89	2.93	3.00
9	2.03	2.12	3.19	2.50	2.67	2.83	2.91	2.91	2.95	3.01
10	2.30	2.32	3.14	2.46	2.56	2.68	2.80	2.80	2.87	2.97
11	2.32	2.37	3.24	2.64	2.70	2.76	2.84	2.84	2.89	2.96
12	2.57	2.61	3.39	2.79	2.84	2.85	2.87	2.87	2.89	2.97
1	2.44	2.48	3.21	2.64	2.69	2.74	2.87	2.87	2.90	2.95
2	2.38	2.41	3.28	2.55	2.60	2.63	2.78	2.78	2.80	2.87
3	2.37	2.44	3.31	2.72	2.81	2.85	2.89	2.89	2.91	2.97
4	2.70	2.79	3.35	3.10	3.16	3.21	3.09	3.09	3.11	3.12
5	2.72	2.77	3.30	2.98	3.04	3.08	3.00	3.00	2.99	3.02
6	2.50	2.57	3.60	2.82	2.83	2.86	2.87	2.87	2.90	2.97
7	2.20	2.32	3.65	2.76	2.87	2.95	2.97	2.97	2.99	3.02
8	2.67	2.78	3.90	3.24	3.32	3.33	3.19	3.19	3.15	3.17
9	2.82	2.94	3.94	3.38	3.45	3.46	3.31	3.31	3.24	3.25
10	3.06	3.09	3.98	3.24	3.28	3.30	3.22	3.22	3.18	3.22
11	3.10	3.15	3.24	3.36	3.41	3.44	3.34	3.34	3.28	3.27
12	3.27	3.31	3.05	3.60	3.70	3.71	3.48	3.48	3.40	3.34
1	3.09	3.13	2.56	3.28	3.33	3.38	3.38	3.38	3.30	3.37
2	3.04	3.12	1.98	3.36	3.39	3.35	3.28	3.28	3.19	3.25
3	3.11	3.18	1.89	3.45	3.49	3.42	3.33	3.33	3.26	3.26
4	3.16	3.22	1.66	3.46	3.50	3.43	3.37	3.37	3.34	3.32
5	2.99	3.09	1.24	3.50	3.57	3.62	3.50	3.50	3.42	3.40
6	3.34	3.43	1.04	3.70	3.78	3.82	3.82	3.82	3.60	3.58
7	3.06	3.26	1.49	3.78	3.86	3.94	3.78	3.78	3.56	3.56
8	3.44	3.59	2.18	3.95	3.98	3.99	3.84	3.84	3.67	3.65
9	3.63	3.73	2.85	3.98	3.99	3.92	3.82	3.82	3.69	3.64
10	3.70	3.79	3.02	4.08	4.10	4.05	3.88	3.88	3.75	3.63
11	3.16	3.19	2.95	3.38	3.53	3.58	3.45	3.45	3.44	3.42
12	3.11	3.09	3.18	3.06	3.06	3.07	3.08	3.08	3.18	3.24
1	2.60	2.59	3.15	2.56	2.61	2.76	2.98	2.98	3.18	3.18
2	1.74	1.82	3.40	2.37	2.64	2.76	2.97	2.97	3.28	3.32
3	1.31	1.50	3.32	2.28	2.46	2.67	3.00	3.00	3.28	3.34
4	1.13	1.31	3.50	2.16	2.33	2.44	2.85	2.85	3.16	3.23
5	0.97	1.06	3.58	1.60	1.98	2.38	2.91	2.91	3.15	3.18
6	0.87	0.93	3.67	1.45	2.07	2.35	2.94	2.94	3.19	3.20
7	1.00	1.16	3.90	1.95	2.35	2.79	3.25	3.25	3.40	3.44
8	1.67	1.84	4.05	2.67	2.93	3.23	3.57	3.57	3.69	3.70
9	2.50	2.62	4.41	3.18	3.40	3.60	3.68	3.68	3.78	3.82
10	2.71	2.81	4.71	3.21	3.32	3.45	3.65	3.65	3.74	3.78

1	2.70	2.92	4.91	3.42	3.59	3.68	3.79	3.79	3.84	3.87
2	2.73	3.09	4.44	3.68	3.82	3.88	3.89	3.89	3.95	3.97
3	2.77	3.02	4.38	3.55	3.65	3.70	3.79	3.79	3.86	3.95
4	2.88	3.21	3.53	3.75	3.89	3.99	3.99	3.99	3.99	3.99
5	2.87	3.18	3.88	3.87	4.02	4.11	4.11	4.11	4.10	4.08
6	2.97	3.43	3.82	3.92	4.06	4.13	4.12	4.12	4.11	4.10
7	3.22	3.70	3.09	4.11	4.23	4.30	4.24	4.24	4.19	4.14
8	2.97	3.68	2.83	4.26	4.39	4.55	4.30	4.30	4.20	4.10
9	4.00	4.32	2.95	4.54	4.59	4.64	4.48	4.48	4.33	4.24
10	3.96	4.59	2.88	4.79	4.78	4.68	4.51	4.51	4.33	4.18
11	4.17	4.30	2.85	4.51	4.66	4.72	4.38	4.38	4.22	4.11
12	4.60	4.68	2.99	4.83	4.85	4.77	4.52	4.52	4.32	4.21
1	4.43	4.86	2.69	4.98	4.98	4.84	4.67	4.67	4.55	4.41
2	3.98	4.36	2.77	4.58	4.69	4.82	4.51	4.51	4.39	4.34
3	4.16	4.33	2.93	4.50	4.58	4.66	4.41	4.41	4.34	4.31
4	2.79	3.15	2.85	3.80	3.90	4.00	4.07	4.07	4.09	4.10
5	2.96	3.50	2.81	4.18	4.35	4.57	4.37	4.37	4.29	4.25
6	3.07	3.50	2.94	4.08	4.23	4.35	4.25	4.25	4.23	4.20
7	2.19	2.70	2.87	3.58	3.78	3.95	3.99	3.99	4.03	4.06
8	2.14	2.50	2.84	3.18	3.33	3.54	3.80	3.80	3.84	3.85
9	2.60	2.84	3.00	2.72	3.37	3.60	3.83	3.83	3.92	3.97
10	2.32	2.75	2.99	3.16	3.39	3.67	3.81	3.81	3.86	3.95
11	2.22	2.53	2.94	3.20	3.43	3.70	3.88	3.88	3.96	3.99
12	2.34	2.67	3.00	3.34	3.60	3.85	3.96	3.96	4.00	4.00
1	2.30	2.46	3.15	2.94	3.22	3.51	3.74	3.74	3.86	3.90
2	2.34	2.52	3.22	3.18	3.42	3.69	3.85	3.85	3.91	3.95
3	2.61	2.77	3.08	3.20	3.40	3.62	3.73	3.73	3.80	3.83
4	2.40	2.65	3.00	3.12	3.30	3.58	3.79	3.79	3.84	3.86
5	2.19	2.50	2.97	3.05	3.24	3.48	3.72	3.72	3.79	3.84
6	2.37	2.56	2.93	3.30	3.49	3.66	3.80	3.80	3.80	3.80
7	2.35	2.60	3.11	3.29	3.53	3.74	3.84	3.84	3.89	3.92
8	2.30	2.60	3.21	3.20	3.53	3.82	3.92	3.92	3.95	3.95
9	2.19	2.63	3.05	3.43	3.69	3.91	3.99	3.99	4.04	4.07
10	2.26	2.64	2.91	3.35	3.55	3.72	3.86	3.86	3.97	4.08
11	2.29	2.57	2.91	3.32	3.52	3.74	3.87	3.87	3.97	4.04
12	2.49	2.76	3.00	3.36	3.59	3.83	3.97	3.97	4.00	4.06
1	2.75	2.99	3.05	3.40	3.63	3.83	4.00	4.00	4.10	4.16
2	2.72	2.98	3.04	3.57	3.75	3.95	4.12	4.12	4.15	4.16
3	2.71	2.89	2.99	3.31	3.49	3.72	4.00	4.00	4.11	4.13
4	2.72	2.85	3.05	3.20	3.36	3.60	3.87	3.87	3.97	3.99
5	2.78	2.86	3.08	3.23	3.40	3.63	3.88	3.88	3.92	3.94
6	2.72	2.78	3.15	3.15	3.34	3.65	3.87	3.87	3.93	3.96
7	2.90	2.97	3.16	3.38	3.55	3.76	3.99	3.99	4.01	4.04
8	2.88	3.00	3.49	3.45	3.59	3.81	4.01	4.01	4.06	4.11
9	2.87	2.96	3.57	3.21	3.38	3.68	3.96	3.96	4.03	4.08
10	2.73	2.87	3.55	3.15	3.36	3.58	3.95	3.95	4.00	4.04
11	2.74	2.79	3.66	3.15	3.37	3.64	3.95	3.95	3.98	4.02
12	2.87	2.92	3.68	3.19	3.38	3.61	3.92	3.92	3.99	4.03
1	2.89	2.96	3.80	3.22	3.37	3.56	3.81	3.81	3.93	3.98
2	2.95	2.99	3.77	3.20	3.37	3.68	3.86	3.86	3.97	4.01
3	2.91	2.95	3.82	3.15	3.36	3.66	3.94	3.94	3.99	4.01
4	2.92	2.97	3.93	3.22	3.41	3.68	3.96	3.96	4.02	4.04
5	2.93	2.98	3.82	3.30	3.50	3.70	3.93	3.93	4.03	4.07
6	3.06	3.10	3.77	3.33	3.53	3.75	3.96	3.96	4.05	4.08
7	3.03	3.07	3.71	3.27	3.58	3.80	3.99	3.99	4.05	4.08
8	3.31	3.39	3.63	3.65	3.75	3.85	3.98	3.98	4.02	4.06
9	3.34	3.46	3.72	3.70	3.80	3.94	4.01	4.01	4.04	4.07
10	3.40	3.48	3.77	3.68	3.80	3.94	4.08	4.08	4.10	4.12
11	3.55	3.59	3.80	3.80	3.89	4.00	4.14	4.14	4.15	4.16
12	3.58	3.62	4.03	3.82	3.90	3.98	4.09	4.09	4.15	4.20
1	3.58	3.65	4.00	3.94	4.00	4.08	4.13	4.13	4.17	4.21
2	3.54	3.62	3.94	3.92	3.98	4.05	4.15	4.15	4.16	4.19
3	3.64	3.75	4.04	3.97	4.03	4.08	4.15	4.15	4.16	4.20
4	3.59	3.79	4.05	4.10	4.18	4.23	4.23	4.23	4.24	4.23

6	3.51	3.53	3.55	3.56	3.57	3.58	3.59	3.60	3.61	3.62	3.63
7	3.52	3.60	3.89	3.87	3.94	4.02	4.14	4.14	4.14	4.14	4.14
8	3.50	3.56	3.95	3.81	3.90	4.03	4.17	4.17	4.18	4.18	4.18
9	3.54	3.62	4.03	3.85	3.92	4.05	4.19	4.19	4.18	4.18	4.18
10	3.60	3.70	4.28	3.90	3.96	4.04	4.16	4.16	4.18	4.18	4.18
11	3.61	3.70	4.28	3.91	3.97	4.03	4.13	4.13	4.16	4.17	4.17
12	3.88	3.98	4.36	4.09	4.10	4.12	4.17	4.17	4.18	4.19	4.19
1	3.88	3.96	4.85	4.02	4.04	4.08	4.19	4.19	4.19	4.20	4.20
2	3.88	3.92	4.81	3.97	4.02	4.08	4.19	4.19	4.18	4.18	4.18
3	4.02	4.03	4.91	4.06	4.08	4.13	4.20	4.20	4.19	4.19	4.19
4	4.03	4.04	4.72	4.07	4.09	4.13	4.17	4.17	4.17	4.17	4.17
5	3.97	3.98	4.84	4.05	4.09	4.14	4.19	4.19	4.18	4.17	4.17
6	3.90	3.96	4.97	4.06	4.09	4.14	4.19	4.19	4.18	4.17	4.17
7	3.87	3.88	4.93	3.95	4.00	4.10	4.19	4.19	4.18	4.17	4.17
8	3.86	3.90	5.08	4.01	4.06	4.13	4.20	4.20	4.19	4.18	4.18
9	3.97	4.00	5.93	4.12	4.18	4.22	4.25	4.25	4.25	4.25	4.25
10	4.10	4.23	5.53	4.30	4.31	4.33	4.34	4.34	4.33	4.31	4.31
11	4.13	4.25	5.44	4.33	4.37	4.39	4.38	4.38	4.35	4.32	4.32
12	4.20	4.30	5.41	4.43	4.47	4.48	4.46	4.46	4.40	4.37	4.37
1	4.55	4.78	4.94	4.92	4.94	4.85	4.59	4.59	4.49	4.47	4.47
2	4.75	4.78	4.62	4.87	4.92	4.93	4.67	4.67	4.58	4.54	4.54
3	4.76	4.85	4.68	5.00	5.03	5.05	4.91	4.91	4.78	4.75	4.75
4	4.62	4.67	4.09	4.78	4.79	4.80	4.61	4.61	4.59	4.58	4.58
5	4.76	4.80	4.00	4.88	4.89	4.90	4.72	4.72	4.63	4.62	4.62
6	4.74	4.84	4.08	5.08	5.08	5.01	4.75	4.75	4.69	4.68	4.68
7	4.66	4.84	4.73	5.10	5.19	5.16	4.93	4.93	4.78	4.75	4.75
8	4.85	4.98	4.80	5.20	5.26	5.26	5.00	5.00	4.82	4.77	4.77
9	5.19	5.68	5.08	6.13	6.15	5.97	5.40	5.40	5.07	4.99	4.99
10	5.44	5.50	5.21	5.48	5.40	5.24	4.85	4.85	4.76	4.74	4.74
11	5.34	5.41	5.41	5.40	5.33	5.22	4.82	4.82	4.66	4.66	4.66
12	5.28	5.35	5.58	5.41	5.40	5.30	5.01	5.01	4.79	4.78	4.78
1	4.91	4.93	5.68	4.94	4.91	4.79	4.68	4.57	4.55	4.54	4.54
2	4.60	4.61	5.30	4.61	4.61	4.60	4.54	4.50	4.44	4.42	4.42
3	4.60	4.65	5.44	4.72	4.74	4.73	4.70	4.67	4.65	4.65	4.65
4	4.19	4.18	5.57	4.15	4.23	4.40	4.45	4.47	4.51	4.54	4.54
5	3.75	3.82	5.88	4.28	4.43	4.58	4.67	4.68	4.71	4.71	4.71
6	3.54	3.85	5.90	4.29	4.47	4.66	4.75	4.83	4.83	4.82	4.82
7	3.88	4.29	5.65	4.92	5.06	5.22	5.22	5.18	5.10	5.03	5.03
8	4.20	4.35	5.31	4.95	5.00	5.09	5.09	5.09	4.99	4.97	4.97
9	4.46	4.90	5.34	5.21	5.32	5.34	5.25	5.19	5.12	5.07	5.07
10	4.45	5.02	5.27	5.29	5.34	5.34	5.30	5.26	5.18	5.11	5.11
11	4.65	5.06	5.58	5.50	5.57	5.64	5.60	5.57	5.46	5.38	5.38
12	5.05	5.52	5.61	5.62	5.66	5.70	5.70	5.70	5.60	5.56	5.56
1	5.13	5.63	6.32	5.72	5.74	5.71	5.68	5.64	5.57	5.50	5.50
2	4.97	5.18	6.30	5.37	5.41	5.47	5.47	5.47	5.37	5.29	5.29
3	5.13	5.30	6.37	5.49	5.51	5.55	5.53	5.47	5.39	5.31	5.31
4	5.27	5.44	6.15	5.62	5.65	5.67	5.67	5.67	5.59	5.49	5.49
5	5.63	5.78	6.09	5.88	5.87	5.80	5.73	5.62	5.47	5.39	5.39
6	5.80	5.87	6.59	5.90	5.89	5.83	5.78	5.63	5.47	5.41	5.41
7	5.45	5.55	7.28	5.69	5.69	5.66	5.60	5.49	5.31	5.28	5.28
8	5.26	5.30	7.52	5.30	5.30	5.30	5.25	5.21	5.12	5.10	5.10
9	5.32	5.32	7.60	5.33	5.34	5.35	5.33	5.26	5.20	5.16	5.16
10	5.26	5.27	7.85	5.29	5.32	5.34	5.34	5.32	5.29	5.27	5.27
11	5.61	5.60	7.48	5.57	5.54	5.50	5.47	5.42	5.40	5.38	5.38
12	5.64	5.63	7.90	5.58	5.58	5.56	5.55	5.55	5.55	5.57	5.57
1	6.38	6.36	8.24	6.29	6.26	6.17	6.12	5.95	5.92	5.90	5.90
2	6.33	6.32	8.11	6.23	6.17	6.07	6.04	6.00	6.00	6.00	6.00
3	6.34	6.37	7.05	6.35	6.33	6.27	6.24	6.15	6.08	6.05	6.05
4	6.13	6.14	6.73	6.17	6.18	6.22	6.25	6.32	6.20	6.05	6.05
5	5.99	6.03	7.49	6.12	6.15	6.25	6.28	6.07	5.92	5.85	5.85
6	6.22	6.53	7.56	6.57	6.57	6.55	6.52	6.41	6.29	6.22	6.22
7	6.22	7.20	7.35	7.22	7.09	6.90	6.70	6.45	6.17	6.17	6.17
8	7.24	7.52	7.06	7.39	7.22	6.85	6.68	6.45	6.17	6.14	6.14
9	7.07	7.37	6.86	7.40	7.23	7.01	6.80	6.57	6.21	6.15	6.15
10	7.32	7.62	6.60	8.00	7.99	7.83	7.62	7.20	6.70	6.56	6.56



1	8.20	8.23	5.15	8.23	8.22	8.05	7.80	7.40	6.81	6.79
2	8.09	8.10	4.31	8.12	8.16	8.16	7.90	7.50	6.84	6.82
3	7.02	7.05	3.76	7.10	7.17	7.25	7.09	6.82	6.41	6.40
4	6.51	6.60	3.84	6.90	7.03	7.20	7.08	6.89	6.48	6.45
5	7.07	7.30	4.57	7.69	7.80	7.87	7.77	7.42	6.90	6.85
6	7.07	7.33	5.07	7.78	7.79	7.76	7.74	7.72	7.42	7.24
7	6.44	6.80	6.01	7.66	7.77	7.78	7.73	7.56	7.15	7.03
8	6.51	6.75	6.11	7.32	7.47	7.52	7.46	7.20	6.75	6.75
9	6.40	6.70	5.29	7.10	7.27	7.45	7.42	7.35	6.90	6.82
10	5.90	6.40	5.20	6.79	6.92	7.11	7.27	7.11	6.74	6.64
11	5.94	6.24	4.66	6.60	6.79	6.99	7.12	7.19	6.80	6.80
12	5.21	5.21	4.70	5.36	5.54	5.98	6.28	6.41	6.22	6.22
1	4.91	5.00	4.35	5.41	5.70	6.00	6.17	6.37	6.30	6.25
2	4.18	4.25	4.20	4.59	5.23	5.73	6.11	5.99	5.97	5.95
3	3.41	3.59	4.29	4.20	4.80	5.39	5.81	6.01	6.11	6.11
4	3.63	3.70	4.94	4.07	4.43	5.04	5.30	5.48	5.73	5.78
5	4.04	4.30	4.50	5.19	5.65	6.05	6.12	5.95	5.98	5.98
6	4.36	4.65	4.55	5.50	5.78	6.20	6.32	6.20	6.13	6.13
7	5.15	5.47	5.00	6.21	6.42	6.72	6.79	6.40	6.30	6.30
8	5.37	5.89	4.79	6.44	6.73	6.96	7.00	6.90	6.30	6.24
9	4.35	4.88	5.40	5.57	5.77	6.08	6.22	6.23	5.94	5.92
10	4.62	5.02	5.62	5.45	5.70	5.96	6.12	5.97	5.80	5.80
11	4.34	4.53	5.50	4.96	5.19	5.63	5.83	5.82	5.75	5.73
12	4.37	4.52	5.42	5.00	5.29	5.75	6.05	5.95	5.79	5.79
1	3.65	4.03	5.62	4.72	5.00	5.47	5.79	5.96	5.81	5.81
2	3.32	3.75	6.08	4.85	5.24	5.67	5.98	6.19	5.90	5.89
3	3.46	3.80	6.40	4.79	5.26	5.62	5.87	6.17	5.85	5.85
4	3.83	4.45	6.92	5.50	5.75	6.03	6.13	6.20	5.98	5.93
5	3.62	4.10	6.68	5.13	5.50	5.86	6.00	6.11	5.98	5.93
6	3.82	4.20	7.10	4.94	5.34	5.70	5.91	6.09	5.81	5.78
7	3.95	4.59	7.88	5.36	5.62	5.92	6.08	6.12	5.86	5.82
8	3.74	4.38	8.81	5.26	5.65	5.97	6.09	6.20	5.73	5.73
9	4.55	4.88	8.60	5.69	5.92	6.22	6.32	6.36	5.70	5.69
10	4.59	5.34	7.81	5.80	5.94	6.15	6.29	6.57	5.85	5.82
11	4.80	5.24	7.33	5.79	5.97	6.13	6.26	6.43	5.73	5.73
12	4.92	5.29	7.65	5.64	5.86	6.03	6.10	6.32	5.59	5.58
1	5.20	5.51	7.21	5.81	5.95	6.09	6.20	6.33	5.83	5.82
2	5.65	5.91	7.10	6.23	6.32	6.41	6.43	6.49	6.85	6.89
3	5.92	6.05	7.15	6.64	6.71	6.70	6.67	6.61	6.88	6.90
4	6.51	7.00	8.30	6.80	6.72	6.72	6.70	6.65	6.85	6.85
5	6.32	6.78	8.99	6.65	6.68	6.69	6.69	6.65	6.88	6.88
6	7.03	7.22	8.92	6.85	6.72	6.69	6.75	6.80	7.03	7.08
7	7.68	7.94	9.05	7.00	6.90	6.82	6.80	6.80	7.09	7.15
8	8.52	8.80	9.09	8.00	7.88	7.73	7.48	7.22	7.57	7.58
9	8.77	8.98	10.07	7.70	7.34	7.17	7.00	7.03	7.31	7.32
10	7.11	7.83	8.11	6.97	6.75	6.71	6.72	6.80	7.02	7.01
11	7.50	7.65	7.89	6.74	6.72	6.72	6.71	6.67	7.17	7.27
12	7.43	8.11	7.34	6.79	6.72	6.71	6.69	6.67	7.11	7.10
1	7.59	7.71	7.07	6.78	6.72	6.71	6.69	6.80	7.30	7.27
2	7.64	7.50	6.14	6.79	6.79	6.82	6.81	6.93	7.38	7.39
3	7.59	7.62	5.86	6.82	6.83	6.90	6.89	6.88	7.49	7.48
4	8.51	8.61	6.16	7.84	7.75	7.64	7.58	7.10	7.80	7.80
5	8.91	9.21	6.62	8.33	8.16	8.10	8.00	7.44	8.12	8.03
6	8.27	8.69	6.04	8.10	8.01	7.98	7.85	7.42	8.06	8.07
7	7.64	8.43	6.78	8.30	8.22	8.21	8.00	7.49	8.06	8.06
8	8.11	9.00	7.20	8.58	8.58	8.55	8.43	7.70	8.32	8.40
9	9.33	9.92	7.55	9.09	8.73	8.60	8.50	8.05	8.51	8.70
10	6.28	7.58	7.72	8.05	8.01	7.99	7.90	7.50	8.39	8.45
11	7.96	8.12	6.09	7.87	7.86	7.85	7.70	7.33	8.00	8.15
12	7.60	7.74	6.52	7.35	7.36	7.45	7.55	7.10	7.75	8.02
1	7.22	7.14	6.31	7.25	7.23	7.20	7.29	7.03	7.65	7.95
2	5.84	6.12	5.48	6.92	6.90	7.20	7.38	7.00	7.64	7.95
3	5.46	5.75	6.05	6.25	6.50	6.90	7.20	6.89	7.60	7.83
4	5.62	5.90	6.06	6.64	6.96	7.39	7.22	6.99	8.01	8.20

7	5.91	6.32	6.43	7.17	7.37	7.62	7.72	7.37	7.97	8.13
8	6.33	6.97	6.11	7.65	7.82	7.91	7.98	7.48	8.09	8.25
9	6.54	7.19	5.85	7.80	7.90	8.10	8.18	7.62	8.36	8.43
10	6.70	7.25	5.70	8.06	8.25	8.35	8.37	8.10	8.48	8.55
11	5.65	5.84	5.43	6.85	7.11	7.50	7.72	7.85	8.02	8.20
12	5.60	6.22	4.91	7.21	7.42	7.71	8.01	7.88	8.12	8.33
1	5.26	5.65	4.82	6.69	7.00	7.38	7.60	7.54	7.83	8.03
2	4.74	5.09	5.50	6.35	6.88	7.40	7.73	7.38	7.85	8.01
3	5.05	5.65	5.52	6.65	7.07	7.35	7.62	7.47	7.80	8.01
4	5.08	5.55	5.37	6.66	6.99	7.35	7.53	7.38	7.90	7.92
5	5.02	5.49	5.54	6.61	6.95	7.34	7.49	7.42	7.95	8.00
6	5.61	6.13	5.71	7.23	7.45	7.68	7.83	7.94	8.12	8.16
7	5.49	5.95	5.70	6.98	7.20	7.54	7.71	7.86	7.98	8.05
8	5.28	5.70	6.17	6.72	6.96	7.38	7.67	7.84	8.01	8.06
9	5.20	5.54	6.33	6.47	6.70	7.11	7.39	7.64	7.85	7.89
10	5.14	5.46	6.62	6.27	6.61	7.02	7.30	7.52	7.78	7.82
11	4.98	5.23	7.10	6.00	6.32	6.77	7.17	7.40	7.73	7.80
12	4.51	4.71	6.90	5.35	5.66	6.11	6.45	7.00	7.48	7.58
1	4.42	4.65	6.93	5.27	5.68	6.16	6.43	6.79	7.16	7.30
2	4.82	5.20	7.22	6.13	6.49	6.91	7.12	7.40	7.68	7.74
3	4.80	5.10	7.28	6.10	6.50	6.99	7.20	7.48	7.74	7.77
4	4.63	4.93	7.46	5.92	6.35	6.89	7.14	7.40	7.69	7.74
5	4.80	5.10	7.67	6.04	6.37	6.88	7.18	7.43	7.68	7.71
6	5.13	5.42	7.90	6.10	6.40	6.82	7.10	7.40	7.65	7.70
7	5.11	5.41	8.37	6.01	6.20	6.61	6.94	7.23	7.50	7.60
8	5.52	5.92	8.34	6.44	6.59	6.95	7.22	7.40	7.62	7.69
9	5.70	6.10	8.46	6.51	6.66	6.89	7.06	7.28	7.51	7.59
10	6.05	6.46	8.76	6.74	6.83	7.05	7.19	7.37	7.61	7.67
11	6.38	6.82	9.93	7.20	7.31	7.45	7.50	7.60	7.80	7.84
12	6.20	6.65	10.07	7.09	7.16	7.31	7.39	7.52	7.75	7.79
1	6.28	6.70	10.55	7.16	7.30	7.48	7.61	7.73	7.95	7.97
2	6.58	7.00	10.12	7.40	7.51	7.68	7.78	7.89	8.09	8.12
3	6.60	7.02	10.35	7.54	7.68	7.83	7.91	8.00	8.16	8.18
4	6.65	7.04	10.14	7.69	7.78	7.91	7.98	8.12	8.28	8.30
5	6.52	7.16	10.37	7.87	7.93	8.04	8.10	8.20	8.31	8.33
6	6.83	7.50	9.87	8.08	8.14	8.23	8.29	8.38	8.47	8.49
7	7.21	7.78	9.34	8.46	8.51	8.49	8.53	8.59	8.63	8.61
8	6.96	7.67	9.68	8.41	8.45	8.42	8.46	8.53	8.56	8.53
9	7.77	8.08	10.35	8.44	8.37	8.40	8.38	8.36	8.38	8.39
10	8.07	8.72	10.78	8.64	8.43	8.47	8.49	8.50	8.57	8.57
11	9.12	9.79	12.90	9.46	9.27	9.00	8.96	8.88	8.86	8.82
12	9.31	9.82	11.98	9.50	9.13	8.84	8.84	8.80	8.73	8.73
1	9.60	10.08	11.89	9.95	9.64	9.28	9.20	9.12	8.96	8.93
2	9.61	9.93	12.32	9.60	9.19	8.94	8.93	8.92	8.84	8.80
3	9.79	10.09	15.27	9.85	9.22	9.25	9.19	9.06	9.09	9.05
4	9.77	10.05	15.68	9.68	9.31	9.18	9.10	9.08	9.01	8.98
5	9.90	9.99	11.14	9.87	9.32	9.29	9.30	9.32	9.21	9.19
6	9.89	10.03	8.78	9.53	9.00	9.01	9.00	9.04	9.05	9.06
7	9.25	9.47	8.42	8.94	8.56	8.67	8.71	8.76	8.78	8.81
8	9.48	9.82	9.26	9.26	9.02	8.86	8.96	8.97	8.91	8.95
9	10.16	10.42	11.09	9.79	9.50	9.32	9.26	9.21	9.06	9.06
10	10.44	10.79	12.24	10.05	9.77	9.46	9.45	9.42	9.29	9.23
11	12.66	13.06	13.67	12.05	11.75	11.12	10.91	10.78	10.54	10.22
12	11.95	12.32	15.04	11.35	10.53	10.45	10.44	10.36	10.08	10.06
1	12.53	12.73	13.97	11.21	10.50	10.35	10.31	10.31	10.10	10.08
2	12.54	12.79	13.92	11.73	11.10	11.09	11.15	11.11	11.11	11.08
3	14.62	15.17	14.74	14.59	13.72	13.48	12.98	12.68	12.27	12.15
4	14.98	15.98	13.01	14.57	13.45	13.25	12.58	12.60	12.40	12.27
5	10.60	11.13	15.33	10.73	10.58	10.73	10.68	10.73	10.81	10.87
6	7.99	8.43	14.46	9.15	9.32	9.68	9.97	10.19	10.31	10.33
7	8.18	8.35	14.96	8.92	9.17	9.46	9.82	9.98	10.01	9.94
8	8.89	9.16	16.10	9.63	9.89	10.08	10.33	10.62	10.68	10.60
9	10.23	10.91	17.13	11.31	11.39	11.58	11.54	11.49	11.53	11.25
10	11.89	12.31	16.64	12.01	11.87	11.81	11.84	11.83	11.83	11.70

1	15.02	14.96	13.31	13.01	12.65	12.57	12.47	12.43	11.96	11.94
2	15.27	14.70	14.05	13.26	12.91	12.73	12.65	12.64	12.40	12.23
3	15.11	15.23	14.29	14.07	13.89	13.89	13.53	13.41	13.15	12.95
4	13.00	12.93	14.29	13.14	13.31	13.33	13.21	13.10	12.89	12.61
5	15.62	15.44	13.71	14.79	14.42	14.38	14.27	14.13	13.88	13.65
6	15.54	14.94	12.93	14.15	14.09	13.74	13.49	13.29	13.10	13.10
7	15.08	15.21	14.32	14.65	14.52	14.25	14.05	13.84	13.64	13.30
8	15.60	16.17	12.07	15.77	15.72	15.27	14.97	14.72	14.26	13.93
9	16.46	17.35	11.10	16.88	16.26	16.07	15.72	15.38	15.12	14.74
10	15.15	16.25	10.24	16.69	16.37	16.18	16.04	15.76	15.58	15.20
11	13.32	14.01	9.19	14.50	14.55	14.71	14.72	14.64	14.69	14.34
12	10.94	11.79	9.31	12.64	13.06	12.96	13.47	13.27	13.58	13.03
1	11.54	12.88	8.66	13.60	13.89	13.89	14.12	13.93	14.05	13.61
2	13.11	13.78	8.92	14.17	14.23	14.09	14.24	14.19	14.17	13.88
3	13.04	14.12	8.61	14.41	14.35	14.11	14.07	13.99	14.05	13.80
4	13.90	14.27	9.31	14.45	14.45	14.30	14.36	14.17	13.86	13.66
5	12.96	13.59	8.70	13.95	13.98	13.74	13.84	13.81	13.55	13.35
6	11.97	12.52	9.46	13.57	13.67	13.78	13.81	13.69	13.63	13.38
7	13.32	14.07	9.60	14.55	14.63	14.51	14.52	14.32	14.06	13.84
8	10.12	11.51	10.49	13.15	13.44	13.55	13.72	13.63	13.61	13.40
9	8.66	10.20	10.47	11.94	12.34	12.72	12.86	12.77	12.55	12.47
10	7.79	9.22	9.77	11.23	11.48	11.68	11.66	11.93	11.58	11.74
11	8.15	8.84	9.74	9.83	10.23	10.53	10.61	11.05	10.75	10.97
12	8.51	9.00	9.91	9.92	10.17	10.47	10.77	10.69	10.98	10.94
1	8.13	8.40	10.04	9.46	9.91	10.21	10.27	10.31	10.65	10.64
2	8.36	8.68	9.76	9.59	9.98	10.44	10.67	10.75	11.08	11.18
3	8.15	8.38	10.20	9.40	9.62	9.92	10.14	10.24	10.62	10.68
4	8.95	9.20	10.73	9.56	10.07	10.42	10.51	10.59	10.82	10.85
5	8.30	8.52	11.06	9.28	9.47	9.91	10.07	10.18	10.47	10.50
6	8.91	9.23	12.06	10.09	10.19	10.49	10.77	10.79	11.10	11.13
7	9.04	9.38	12.24	10.23	10.43	10.80	10.85	10.89	11.15	11.13
8	9.58	10.07	11.80	11.02	11.20	11.68	11.73	11.80	11.98	11.91
9	9.56	10.10	11.86	11.10	11.42	11.71	11.88	11.92	12.03	11.96
10	9.00	9.36	11.39	10.48	10.71	11.15	11.34	11.39	11.60	11.46
11	8.81	9.28	10.22	10.58	10.91	11.35	11.58	11.71	11.92	11.80
12	9.17	9.56	9.73	10.61	10.89	11.40	11.51	11.58	11.77	11.64
1	9.26	9.67	9.19	10.82	11.07	11.52	11.80	11.76	11.98	11.84
2	9.19	9.50	9.15	10.54	10.78	11.31	11.54	11.59	11.77	11.73
3	9.47	9.89	9.69	10.99	11.24	11.80	11.99	12.04	12.19	12.16
4	9.98	10.42	9.48	11.50	11.73	12.19	12.39	12.43	12.45	12.47
5	10.07	10.60	8.97	11.85	12.07	12.51	12.71	12.78	12.84	12.82
6	10.08	11.25	7.92	12.92	13.19	13.65	13.73	13.78	13.70	13.71
7	10.26	11.15	7.65	13.14	13.38	13.71	13.81	13.83	13.77	13.68
8	10.81	11.37	8.06	12.53	12.63	12.78	12.87	12.85	12.89	12.80
9	11.01	11.44	8.02	12.45	12.55	12.74	12.82	12.76	12.63	12.52
10	10.58	11.04	7.93	12.02	12.22	12.44	12.50	12.40	12.31	12.27
11	9.33	9.78	7.91	11.04	11.28	11.50	11.69	11.69	11.66	11.57
12	8.68	9.14	7.81	10.47	10.86	11.25	11.53	11.50	11.68	11.59
1	8.08	8.64	7.57	9.99	10.49	11.09	11.47	11.45	11.63	11.58
2	8.34	8.63	7.75	9.99	10.29	10.80	11.12	11.07	11.30	11.19
3	8.82	9.31	7.09	10.70	11.08	11.56	11.88	11.87	12.11	11.96
4	8.44	9.03	6.52	10.43	10.81	11.32	11.68	11.63	11.85	11.71
5	8.11	8.55	6.47	9.89	10.30	10.90	11.31	11.32	11.62	11.54
6	7.30	7.55	6.90	8.97	9.30	9.78	10.17	10.20	10.64	10.68
7	7.01	7.31	6.18	8.62	9.09	9.66	10.14	10.15	10.55	10.59
8	7.47	7.78	6.11	8.99	9.48	9.98	10.41	10.49	10.86	10.84
9	7.36	7.68	5.71	8.95	9.32	9.87	10.22	10.32	10.70	10.65
10	7.27	7.57	5.51	8.83	9.27	9.78	10.17	10.33	10.78	10.66
11	7.41	7.66	5.67	8.63	8.96	9.53	9.84	10.01	10.44	10.38
12	7.36	7.58	5.78	8.43	8.68	9.20	9.45	9.62	10.02	9.87
1	7.24	7.41	5.82	7.98	8.21	8.58	8.83	8.98	9.45	9.34
2	7.48	7.64	5.91	8.16	8.35	8.69	8.98	9.16	9.55	9.35
3	6.90	6.97	5.86	7.32	7.40	7.65	7.85	8.03	8.29	8.21
4	6.37	6.46	6.03	6.78	6.93	7.17	7.27	7.40	7.68	7.53

7	6.08	6.08	6.62	6.67	6.86	7.12	7.21	7.27	7.48	7.48
8	5.95	6.01	6.90	6.54	6.70	7.03	7.22	7.35	7.80	7.79
9	5.42	5.54	7.66	6.27	6.52	6.89	7.17	7.35	7.95	7.85
10	5.15	5.32	7.92	6.09	6.35	6.70	7.05	7.25	7.80	7.74
11	5.37	5.45	6.83	6.20	6.40	6.71	7.05	7.22	7.77	7.65
12	5.61	5.67	7.03	6.21	6.35	6.61	6.90	7.07	7.61	7.55
1	5.70	5.74	6.98	6.23	6.40	6.71	6.97	7.11	7.58	7.57
2	5.81	5.84	6.62	6.38	6.52	6.75	7.03	7.23	7.73	7.65
3	5.62	5.65	6.55	6.32	6.47	6.70	6.95	7.15	7.60	7.56
4	5.64	5.88	6.89	6.54	6.78	7.05	7.31	7.50	7.91	7.89
5	5.78	6.24	7.12	7.52	7.81	8.05	8.30	8.41	8.79	8.66
6	5.93	6.30	7.57	7.72	7.98	8.24	8.46	8.60	8.98	8.78
7	5.87	6.02	7.43	7.34	7.60	7.90	8.12	8.27	8.50	8.40
8	6.30	6.45	7.86	7.69	7.95	8.33	8.58	8.76	9.14	9.00
9	6.65	6.87	8.25	8.33	8.63	8.94	9.19	9.38	9.61	9.51
10	6.92	7.20	8.10	8.60	8.89	9.26	8.51	9.66	9.92	9.76
11	5.61	6.30	8.03	7.68	8.02	8.42	8.76	9.00	9.29	9.11
12	5.71	6.50	9.09	7.76	8.05	8.47	8.80	9.00	9.30	9.15
1	5.83	6.38	9.09	7.67	7.91	8.28	8.55	8.71	8.90	8.84
2	5.89	6.30	9.01	7.18	7.42	7.75	8.03	8.18	8.38	8.36
3	5.77	6.09	9.32	7.05	7.25	7.61	7.86	8.11	8.42	8.31
4	6.06	6.44	9.56	7.55	7.74	8.17	8.49	8.68	9.01	8.89
5	6.24	6.61	9.26	7.76	7.95	8.37	8.67	8.91	9.21	9.13
6	6.75	7.12	8.84	8.16	8.35	8.68	8.93	9.11	9.27	9.21
7	6.79	7.01	8.03	7.96	8.08	8.38	8.59	8.75	8.89	8.82
8	7.13	7.45	7.55	8.35	8.50	8.71	8.93	9.08	9.21	9.19
9	7.56	7.87	8.39	8.68	8.82	8.89	9.12	9.25	9.32	9.32
10	7.53	7.81	8.45	8.39	8.46	8.57	8.72	8.82	8.98	8.95
11	7.65	7.85	7.90	8.22	8.30	8.38	8.53	8.66	8.81	8.76
12	8.51	8.69	7.65	9.20	9.22	9.23	9.25	9.22	9.20	9.09
1	8.51	8.69	7.79	9.20	9.22	9.23	9.25	9.22	9.20	9.09
2	8.74	8.91	8.04	9.11	9.12	9.07	9.03	9.00	8.92	8.84
3	9.05	9.14	8.14	9.47	9.36	9.37	9.32	9.27	9.19	9.10
4	9.25	9.46	8.26	9.64	9.58	9.46	9.35	9.24	9.16	9.08
5	8.93	9.09	8.47	9.32	9.28	9.16	9.13	9.11	9.04	8.97
6	9.00	8.83	8.04	8.84	8.79	8.68	8.66	8.63	8.62	8.61
7	8.21	8.00	8.04	7.99	7.99	8.02	8.12	8.14	8.14	8.13
8	7.94	7.69	7.63	7.47	7.50	7.49	7.59	7.74	7.81	7.87
9	8.26	8.29	7.75	8.48	8.43	8.32	8.33	8.28	8.25	8.22
10	8.24	8.29	7.62	8.44	8.44	8.33	8.35	8.27	8.24	8.21
11	8.13	8.00	7.32	7.87	7.92	7.88	7.93	7.92	7.92	7.92
12	7.85	7.74	7.28	7.73	7.70	7.75	7.78	7.81	7.85	7.88
1	7.91	7.91	7.79	7.87	7.89	7.91	7.99	7.96	7.99	8.01
2	8.06	8.10	8.04	8.23	8.30	8.32	8.35	8.41	8.43	8.44
3	8.11	8.12	8.14	8.47	8.49	8.49	8.56	8.53	8.54	8.55
4	8.07	8.17	8.26	8.59	8.61	8.64	8.67	8.64	8.64	8.63
5	8.19	8.42	8.47	9.00	8.99	9.03	9.03	9.03	9.01	8.99
6	7.98	7.99	8.04	8.33	8.38	8.43	8.50	8.48	8.48	8.48
7	8.07	8.02	8.04	8.24	8.29	8.36	8.45	8.43	8.43	8.43
8	7.76	7.64	7.63	7.81	7.91	8.03	8.19	8.27	8.32	8.36
9	7.67	7.74	7.75	8.07	8.25	8.54	8.77	8.91	8.98	9.05
10	7.43	7.50	7.62	7.96	8.11	8.42	8.64	8.75	8.82	8.90
11	7.33	7.37	7.32	7.66	7.84	8.14	8.40	8.54	8.62	8.69
12	7.32	7.34	7.28	7.52	7.63	7.88	8.12	8.20	8.27	8.34

```
program RandomRate (input,output);
```

```
uses Printer;
```

```
var Rates = array [0..30] of real;
```

```
    Arr = array [1..100] of real;
```

```
    Freq = array [0..400] of integer;
```

```
    Strig = string[14];
```

```
var i,j,Seed,Num,Count:integer;
```

```
    T1,T2,T5,T7,T10,T20,T1S,T1L:Rates;
```

```
    AccumT,AccumL,AccumS,AnnuityL,AnnuityS,AnnuityT,
```

```
        AvgRateT,AvgRateL,AvgRateS:Arr;
```

```
    ModFlag:boolean;
```

```
    Dist,Tot3,Tot2,TotL:freq;
```

```
    Outfile:text;
```

```
    TotA,TotP,Rate:real;
```

```
var print = false;
```

```
    output = false;
```

```
    FileName = 'a:accums.p02';
```

```
function Random2 (var Seed: integer): real;
```

```
    (Generate a psuedo-random number such that 0<=Random<1)
```

```
    (page 116 Cooper's Condensed Pascal)
```

```
var Temp:real;
```

```
const MODULUS = 65536;(These are 'magic' numbers)
```

```
    MULTIPLIER = 25173;    (that produce a psuedo-random)
```

```
    INCREMENT = 13849;(sequence of numbers)
```

```
begin
```

```
    (writeln ('Seed=',Seed:1);)
```

```
    Seed:=(MULTIPLIER*Seed)+INCREMENT) mod MODULUS;
```

```
    (Pick an integer from 0 to MODULUS-1)
```

```
    (writeln ('Seed=',Seed:1);)
```

```
    Temp:=Seed/MODULUS;
```

```
    (Adjust it to fall between 0 and 1)
```

```
    writeln ('Random 0-1 =',Temp:5:5);
```

```
    Random2:=Temp;
```

```
end;
```

```
function Z(var Seed:integer):real;
```

```
    (produces random variable from normal distribution)
```

```
var U1,U2,Temp1,Temp2:real;
```

```
const pi=3.141592654;
```

```
begin
```

```
    U1:=Random;
```

```
    (writeln ('U1=',U1:5:5);)
```

```

random;
writeln ('U2=',U2:5:5);)
U:=sqrt((-2)*ln(U1));
U:=cos(2*pi*U2);
Temp1*Temp2;
writeln ('Z=',Temp1*Temp2:7:5);)
end function Z)

function Revert(T1,TInfinity:real):real;
var Left,Right,Min,Max:real;

Left:=0.015* ((TInfinity-T1)*(TInfinity-T1)*(TInfinity-T1));
writeln ('left=',left);)
Right:=0.5 * (TInfinity-T1);
writeln ('right=',right);)
if Left<Right then
begin
Min:=Left;
Max:=Right;
(writeln ('Min=Left');)
end
else
begin
Min:=Right;
Max:=Left;
(writeln ('Min=Right');)
end;
if Left<TInfinity then
begin
Revert:=Min;
(writeln ('Revert=Min=',Min:8:8);)
end
else
begin
Revert:=Max;
(writeln ('Revert=Max=',Max:8:8);)
end;
end function Revert)

function Revert2(T1,TInfinity:real):real;
var Left,Right,Min,Max:real;

if (T1>4.0) and (T1<10.0) then Revert2:=0
else
Left:=0.015* ((TInfinity-T1)*(TInfinity-T1)*(TInfinity-T1));
writeln ('left=',left);)
Right:=0.5 * (TInfinity-T1);

```

```

writeln ('right=',right);)
if Left<Right then
begin
  Min:=Left;
  Max:=Right;
  (writeln ('Min=Left');)
end
else
begin
  Min:=Right;
  Max:=Left;
  (writeln ('Min=Right');)
end;
if T1<TInfinity then
begin
  Revert2:=Min;
  (writeln ('Revert2=Min=',Min:8:8);)
end
else
begin
  Revert2:=Max;
  (writeln ('Revert2=Max=',Max:8:8);)
end;
; (Revert)

procedure Lattice (Num:integer; T1Pres:real; var T1:rates);
var
  i:integer;
  K:real;
begin
  T1[0]:=T1Pres;
  for i:=1 to Num do
  begin
    K:=Random;
    K:=round(K);
    if K>0.5 then T1[i]:=T1[i-1]-0.5
    else
      T1[i]:=T1[i-1]+0.5;
  end;

procedure Second (Num:integer; T1Pres:real; var T1:rates);
var
  VolFact = 0.27;
  i:integer;
begin
  T1[0]:=T1Pres;
  for i:=1 to Num do
  begin

```

```

e:=exp(Z(Seed)*VolFact);
T1[i]:=T1[i-1]*e;
end;

```

```

procedure Third (Num:integer; T1Pres,T20Pres:real;
var T1,T2,T5,T7,T10,T20:rates; ModFlag:boolean);

```

```

VolFact = 0.23;

```

```

AnticRate,a,b,sd20,Temp:real;

```

```

T1[0]:=T1Pres;

```

```

T20[0]:=T20Pres;

```

```

T2[0]:=0;

```

```

T5[0]:=0;

```

```

T7[0]:=0;

```

```

T10[0]:=0;

```

```

for i:=1 to Num do

```

```

e:=exp(Z(Seed)*VolFact);

```

```

(writeln ('e=',e:7:4); )

```

```

if not ModFlag then

```

```

T1[i]:=(T1[i-1]+Revert(T1[i-1],T1[0]))*e

```

```

else

```

```

T1[i]:=(T1[i-1]+Revert2(T1[i-1],T1[0]))*e;

```

```

(Get 20-year rate)

```

```

if T1[i]<=0.10 then

```

```

begin

```

```

a:=0.8;

```

```

b:=2.5;

```

```

end

```

```

else

```

```

begin

```

```

a:=0.6;

```

```

b:=4.5;

```

```

end;

```

```

AnticRate:=(a*T1[i])+b;

```

```

if AnticRate<=0.10 then

```

```

sd20:=0.2+(0.1*AnticRate)

```

```

else

```

```

sd20:=1.2;

```

```

Temp:=Z(Seed)*sd20;

```

```

(writeln ('add-on:',Temp:7:7);)

```

```

T20[i]:=AnticRate+Temp;

```

```

(Interpolate for intermediate rates)

```

```

T2[i]:=(0.64*T1[i])+(0.36*T20[i]);

```

```

T5[i]:=(0.39*T1[i])+(0.61*T20[i]);

```

```

T7[i]:=(0.24*T1[i])+(0.76*T20[i]);

```

```

T10[i]:=(0.16*T1[i])+(0.84*T20[i]);

```



```

1: (OneYearT)
2:
3: function Power(Base:real; n:integer):real;
4:
5:   temp:real;
6:   i:integer;
7:
8: begin
9:   temp:=Base;
10:  for i:=2 to n do
11:    temp:=temp*Base;
12:  Power:=temp;
13: end;
14:
15: function SumAnn(i:real; n:integer):real;
16:
17:   Temp,d:real;
18:
19: begin
20:   d:=(i/100)/(1+(i/100));
21:   Temp:=Power(1+(i/100),n);
22:   SumAnn:=100*(Temp-1)/d;
23: end;
24:
25: procedure Bisect(Num:integer; Total:real; var i:real);
26:
27:   a,b,p,fa,fp:real;
28:   t:integer;
29:   done:boolean;
30:
31: begin
32:   (writeln ('Total=',Total:5:5);)
33:   a:=2.0;
34:   b:=30.0;
35:   t:=0;
36:   done:=false;
37:   while not done do
38:   begin
39:     p:=a+((b-a)/2);
40:     fp:=SumAnn(p,Num);
41:     (writeln ('p=',p:4:4,' SumAnn=',fp:4:4);)
42:     fa:=SumAnn(a,Num);
43:     if abs(fp-Total)<0.0001 then done:=true;
44:     t:=t+1;
45:     if (fp-Total)*(fa-Total)>0 then a:=p
46:       else b:=p;
47:     if t>50 then done:=true;
48:   end;
49:   (writeln ('p=',p:5:5);)
50:   i:=p;
51: end;
52:
53: procedure FinOut (Num:integer; T1:rates; var Payout:freq;
54:   var Accum, Periodic, Intrate:real);

```

```

Place:=integer;

in
  Accum:=100;
  Periodic:=100;
  for i:= 0 to 400 do
    begin
      Payout[i]:=0;
    end;
  for i:=1 to Num do
    begin
      Accum:=Accum*(1+(T1[i]/100));
      Periodic:=100+(Periodic*(1+(T1[i]/100)));
      Place:=round(T1[i]*10);
      Payout[Place]:=Payout[Place]+1;
    end; (i)
  Periodic:=Periodic-100;
  Bisect (Num,Periodic,IntRate);
end;

in (RandRate)
  Randomize;
  Count:=0;
  Seed:=25;
  Num:=20;
  ModFlag:=false;
  for t:=0 to 400 do
    begin
      Tot2[t]:=0;
      Tot3[t]:=0;
      TotL[t]:=0;
    end;
  for j:=1 to 100 do
    begin
      writeln ('Generating Scenario: ',j:1);

TD)
      Third (Num,8.0,8.5,T1,T2,T5,T7,T10,T20,ModFlag);
      FinOut (Num,T1,Dist,TotA,TotP,Rate);
      for t:=0 to 400 do
        Tot3[t]:=Tot3[t]+Dist[t];
      AccumT[j]:=TotA;
      AnnuityT[j]:=TotP;
      AvgRateT[j]:=Rate;

COND)
      Second (Num,8.0,T1S);
      FinOut (Num,T1S,Dist,TotA,TotP,Rate);
      for t:=0 to 400 do
        Tot2[t]:=Tot2[t]+Dist[t];
      AccumS[j]:=TotA;
      AnnuityS[j]:=TotP;
      AvgRateS[j]:=Rate;

```

ATTICE)

```
Lattice (Num,8.0,T1L);  
FinOut (Num,T1L,Dist,TotA,TotP,Rate);  
for t:=0 to 400 do  
begin  
  TotL[t]:=TotL[t]+Dist[t];  
  Count:=Count+Dist[t];  
end;  
AccumL[j]:=TotA;  
AnnuityL[j]:=TotP;  
AvgRateL[j]:=Rate;
```

end; (j)

PRINT OUT RESULTS)

```
assign (Outfile,FileName);  
rewrite (Outfile);
```

INTEREST RATE DISTRIBUTIONS)

```
if false then begin  
writeln ('Third Second Lattice');  
for t:=0 to 400 do  
begin  
  writeln ((t/10):4:1,' : ',Tot3[t]:3,' ',Tot3[t]/Count:3:2,' ',  
    Tot2[t]:3,' ',Tot2[t]/Count:3:2,' ',  
    TotL[t]:3,' ',TotL[t]/Count:3:2,' ');  
  writeln (Outfile,(t/10):4:1,' ',Tot3[t]:3,' ',Tot3[t]/Count:3:2,' ',  
    Tot2[t]:3,' ',Tot2[t]/Count:3:2,' ',  
    TotL[t]:3,' ',TotL[t]/Count:3:2,' ');  
end;  
end;
```

end;

OUTPUT DISTRIBUTIONS)

```
for j:=1 to 100 do  
begin  
  writeln (AccumT[j]:8:2,' ',AccumS[j]:8:2,' ',AccumL[j]:8:2,' ',  
    AnnuityT[j]:8:2,' ',AnnuityS[j]:9:2,' ',AnnuityL[j]:8:2,' ',  
    AvgRateT[j]:5:2,' ',AvgRateS[j]:5:2,' ',AvgRateL[j]:5:2);  
  writeln (Outfile,AccumT[j]:8:2,' ',AccumS[j]:8:2,' ',AccumL[j]:8:2,' ',  
    AnnuityT[j]:8:2,' ',AnnuityS[j]:9:2,' ',AnnuityL[j]:8:2,' ',  
    AvgRateT[j]:5:2,' ',AvgRateS[j]:5:2,' ',AvgRateL[j]:5:2);  
end;  
close (Outfile);  
writeln ('Count=',Count:5);  
p. (RandRate)
```